Stacking Moxon Rectangles
Part 1: Vertically Stacking Horizontally Oriented Rectangles

L. B. Cebik, W4RNL (SK)

The Moxon rectangle is a 2-element array using dual coupling between elements to produce its nearly cardioidal pattern. Because it depends upon both the mutual coupling between parallel portions of the elements and the coupling between element ends, it is not amenable to the addition of further elements for increased gain. In other words, a Moxon rectangle is not expandable by the addition of director in the manner of a standard Yagi.

An alternative route to increased gain is the stacking of like antennas and feeding the antennas in phase. At HF, operators employ stacks for a number of tasks. Combined, the antennas deliver the highest gain available. However, separately, the two antennas in the stack exhibit different take-off (TO) angles--or elevation angles of maximum radiation. Having a choice among the potentials allows the operator to elect an elevation most suited to a given propagation path. At VHF, additional gain is normally used solely for the purpose of increased gain.

Stacking Moxon rectangles is certainly possible. However, implementing that possibility requires that we answer several questions:

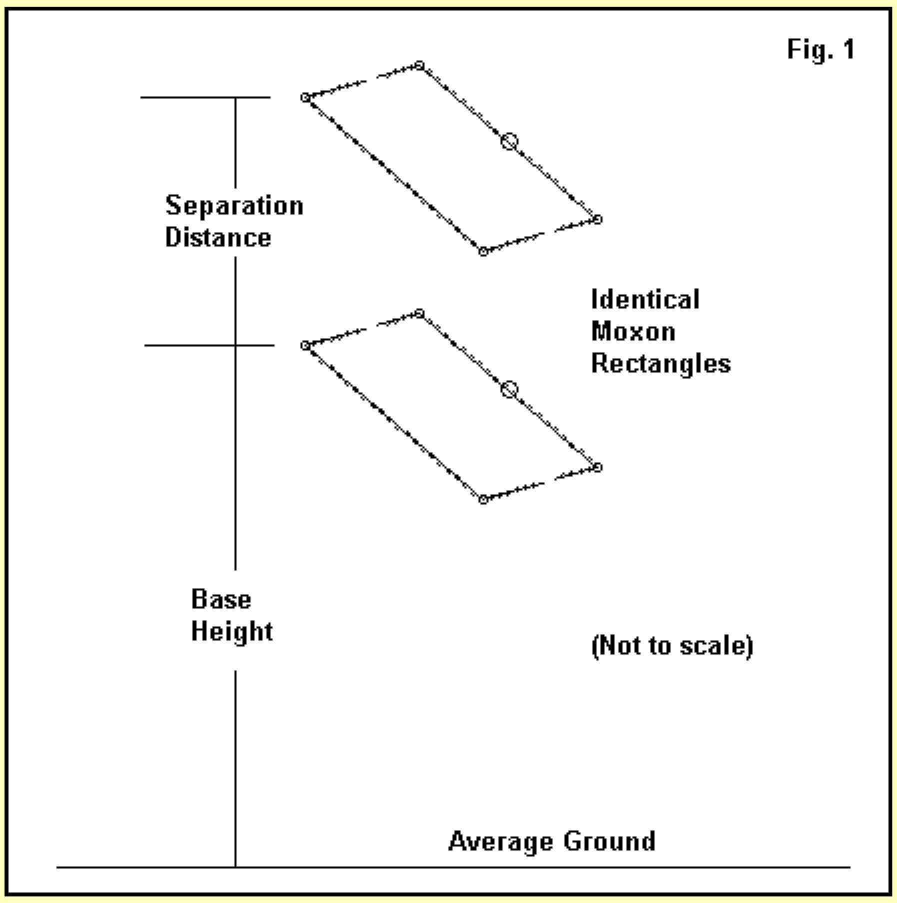
1. What performance can we obtain from a stack of at least 2 Moxon rectangles--and is it sufficient to justify the added mechanical and electrical requirements of such a system?

2. Just how does stacking work?

To answer these questions, we shall divide the stacking question into two parts: the vertical stacking of horizontally oriented rectangles and the vertical stacking of vertically oriented rectangles. Since the latter type of stack is normally used only at VHF and above, we shall begin with the more general case.

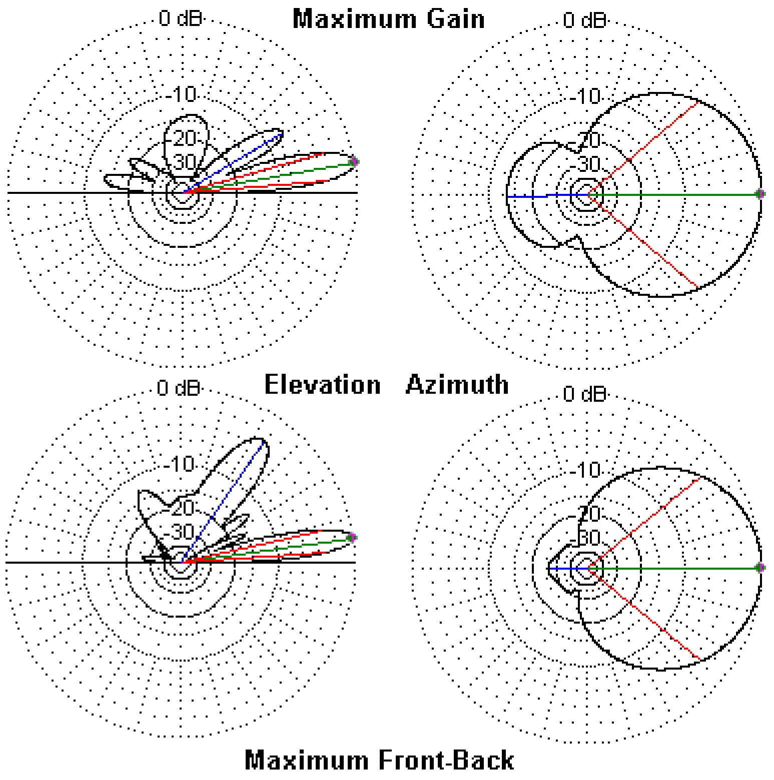
A VHF Stack of 2 Horizontal Moxon Rectangles

Fig. 1 shows the general outline of a 2-stack of (or stack of 2) Moxon rectangles. We normally begin for convenience with two identical antennas, although it is not at necessary that we do this. With sufficient patience, we might customize each antenna for its position in the stack. However, the added benefits of such tedious work rarely outweigh the design and construction effort.



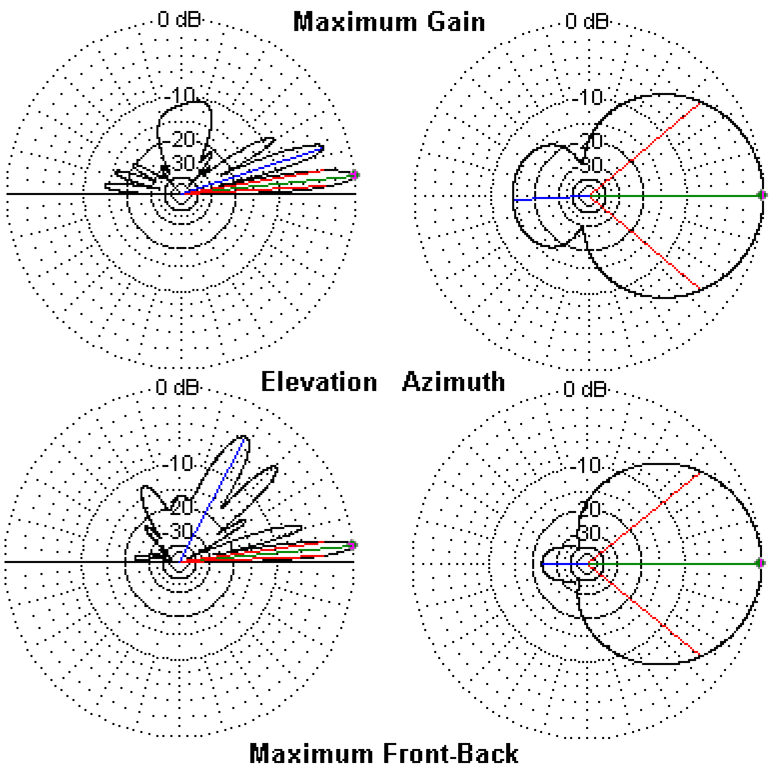
We must decide upon a base height for the array. Normally, we decide the base height of the lower antenna in the stack based upon task specifications and constraints. Since this discussion aims for some general ideas rather than a task-specific design, we shall arbitrarily select base heights of about 1 wavelength and about 2 wavelengths for the exercise. These heights translate at 146 MHz--the center of the 2-meter amateur band--into about 80" (6.67') and 160" (13.33'), respectively. Above a base height of 2 wavelengths, about the only parameter of operation that will change is the TO angle. Hence, the selection gives us a fair representation of performance.

The next decision involves the distance between the lower and the upper antennas in the array. We shall have more to say about this variable at the end of our Moxon discussion. However, in general terms, arrays with low element counts tend to show that the separation required for maximum gain from the array and the separation required for retention of the high front-to-back ratio that is a hallmark of the Moxon rectangle are not the same distances. In fact, they are so far apart that we obtain very different patterns for the two conditions.



El/Az Plots: Moxon Stack
1-Wavelength Base Height
Fig. 2

With a base height of 1 wavelength, as shown in the patterns in **Fig. 2**, the maximum gain pattern shows a considerable vertical elevation lobe that results from spacing that is just above a half-wavelength. The maximum front-to-back patterns result from a separation of just over 1 wavelength. The azimuth patterns, taken at the TO angle, show the differential of front-to-back ratio, but do not reveal the forward gain differential of just over 1 dB. The elevation angle for the maximum gain configuration appears normal, that is, similar to the pattern of a single array at a height that is about 2/3 the way between the two physical antennas. However, the maximum front-to-back stack shows oddities that do not appear with a single antenna, including a high variability in the strength of secondary elevation lobes. The number of lobes is a function of the added height of the top antenna and the fact the that elevation structure is a function of the lobes produced by both antennas. Since the radiation combines at a distance from the pair of antennas and since it also includes ground reflections as well as direct radiation, the pattern of radiation lobes and nulls can be erratic to the eye, even if predictable and calculable in straightforward ways.



El/Az Plots: Moxon Stack
2-Wavelength Base Height
Fig. 3

Fig. 3 repeats the same modeling exercise using a 2-wavelength base height. Between the azimuth patterns in this set and the one for a 1-wavelength height, there is little to choose except for a slight addition to the forward gain for the higher pair. However, the elevation patterns, even for the maximum gain condition, begin to show some of the variations in lobe strength that we saw only in the maximum front-to-back pattern at a 1-wavelength base height. We find more elevation lobes as a simple function of the greater overall height of the antennas. However, on either side of the most vertical lobe, we see some "suppressed" lobes, that is lobes that are weaker than we might expect from a single antenna. These are functions of the complex combinations of direct and reflected radiation from each of the antennas composing the stack.

The maximum front-to-back elevation pattern is especially interesting. It breaks the single large forward secondary lobe into 2 lobes, with additional strong and weak lobes, relative to the comparable pattern in **Fig. 2** for a 1-wavelength base height. We can see that the stacked array offers strong radiation either at very low or very high elevation angles.

Now let's overlay the patterns in our minds and note the similarities of corresponding elevation patterns for both base-height levels. In the maximum front-to-back elevation pattern, we note a low angle main lobe, at least one strong lobe at mid-elevation angles, and a moderate strength vertical lobe--or perhaps "bubble." The maximum gain patterns for each base-height level show increasing radiation strength below about 35 degrees, with a single vertical lobe of considerable strength. These pattern similarities are more than accidental.

As we increase the height of an antenna or a complex array, such as our stacks, we find that elevation pattern properties tend to repeat themselves every 1/2 wavelength, allowing for the addition of new lobes with every significant height increase. Since the pattern pairs in **Fig. 3** are almost exactly 1 wavelength higher than those in **Fig. 2**, we should expect to see the overall outline of the pattern repeated. The phenomenon is perfectly general. Hence, you may begin with an antenna at any height above about 1/2 wavelength and then check the patterns at half-wavelength intervals above that. A single antenna will do for such a modeling exercise, but the principle applies as well to complex arrays.

A More Detailed Analysis of the VHF Arrays

The azimuth and elevation patterns sample the stack performance at only two points of separation for each base height. It is useful to examine data for the entire span of possible separations for the two antennas in the stack. Therefore, I modeled each stack at 5" intervals of separation, always leaving the lower antenna at its base height. 5" at 146 MHz is just a little under 1/16 wavelength, so the accumulated data gives us picture of array performance for regular intervals. The following tables summarize the data recorded for each new amount of separation.

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2-Meter Moxon 2-Stack: 146 MHz: Base Height: 80" (Approx. 1 wl)

Single Antenna	Gain dBi	Front-Back Ratio dB	TO Angle degrees	Feed Z R+/-jX Ohms
	11.29	26.15	13.7	56 + j 3

Separation In.	WL	Gain dBi	Front-Back Ratio dB	TO Angle degrees	Top Z R+/-jX Ohms	Bottom Z R+/-jX Ohms
35	0.4277	13.68	13.63	11.1	65 + j 10	69 + j 10
40	0.4888	14.08	13.22	10.7	69 + j 6	70 + j 7
45	0.5498	14.44	13.08 -	10.4	71 - j 1	72 + j 2
50	0.6109	14.65 +	13.42	10.1	68 - j 10	71 - j 5
55	0.6720	14.62	14.41	9.6	60 - j 14	65 - j 11
60	0.7331	14.41	16.05	9.3	52 - j 12	57 - j 11
65	0.7942	14.14	18.20	9.0	48 - j 9	52 - j 9
70	0.8553	13.89	20.62	8.9	46 - j 5	49 - j 5
75	0.9164	13.67	23.06	8.7	46 - j 2	48 - j 2
80	0.9775	13.48	25.06	8.4	47 + j 1	48 + j 1
85	1.0386	13.32	26.05 +	8.1	48 + j 2	49 + j 3
90	1.0997	13.18	25.88	7.8	49 + j 3	50 + j 5
95	1.1608	13.08	24.95	7.7	50 + j 4	51 + j 7

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2-Meter Moxon 2-Stack: 146 MHz: Base Height: 160" (Approx. 2 wl)

Single Antenna	Gain dBi	Front-Back Ratio dB	TO Angle degrees	Feed Z R+/-jX Ohms
	11.66	30.23	7.1	54 + j 2

Separation In.	WL	Gain dBi	Front-Back Ratio dB	TO Angle degrees	Top Z R+/-jX Ohms	Bottom Z R+/-jX Ohms
35	0.4277	14.16	13.51	6.3	66 + j 10	68 + j 10
40	0.4888	14.60	13.21	6.2	69 + j 7	70 + j 7
45	0.5498	15.00	13.11 -	6.1	72 - j 0	72 + j 1
50	0.6109	15.26	13.38	5.9	70 - j 9	70 - j 7
55	0.6720	15.30 +	14.14	5.9	61 - j 15	64 - j 12
60	0.7331	15.14	15.44	5.8	53 - j 14	56 - j 13
65	0.7942	14.90	17.15	5.7	48 - j 10	50 - j 10
70	0.8553	14.67	19.02	5.7	46 - j 6	47 - j 6
75	0.9164	14.47	20.79	5.6	45 - j 2	46 - j 2
80	0.9775	14.30	22.18	5.4	46 + j 0	46 + j 1
85	1.0386	14.17	23.01	5.4	47 + j 2	47 + j 3
90	1.0997	14.06	23.21 +	5.2	48 + j 4	48 + j 5
95	1.1608	13.98	22.94	5.1	49 + j 5	50 + j 7

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For each base height, I have listed the corresponding properties of a single Moxon rectangle. The forward gain improves with added height, but not by a major amount. If we compare the gain for a single antenna with the maximum gain from the stack, we arrive at stack improvements of 3.33 and 3.64 dB, respectively for the lower and the higher stack. There is a generalization that the maximum gain theoretically obtainable by stacking arrays is 3 dB, and the reality of material losses dictates that the actual realized gain advantage will be slightly less than 3 dB in an optimized stack. The excess gain for our calculations in not erroneous, but rather stems from comparing a stack with an antenna at the lower of the two stack levels. Since the stack has a composite height advantage of about 2/3 the distance between antennas in the stack, the excess is natural as a function of the greater effective stack height.

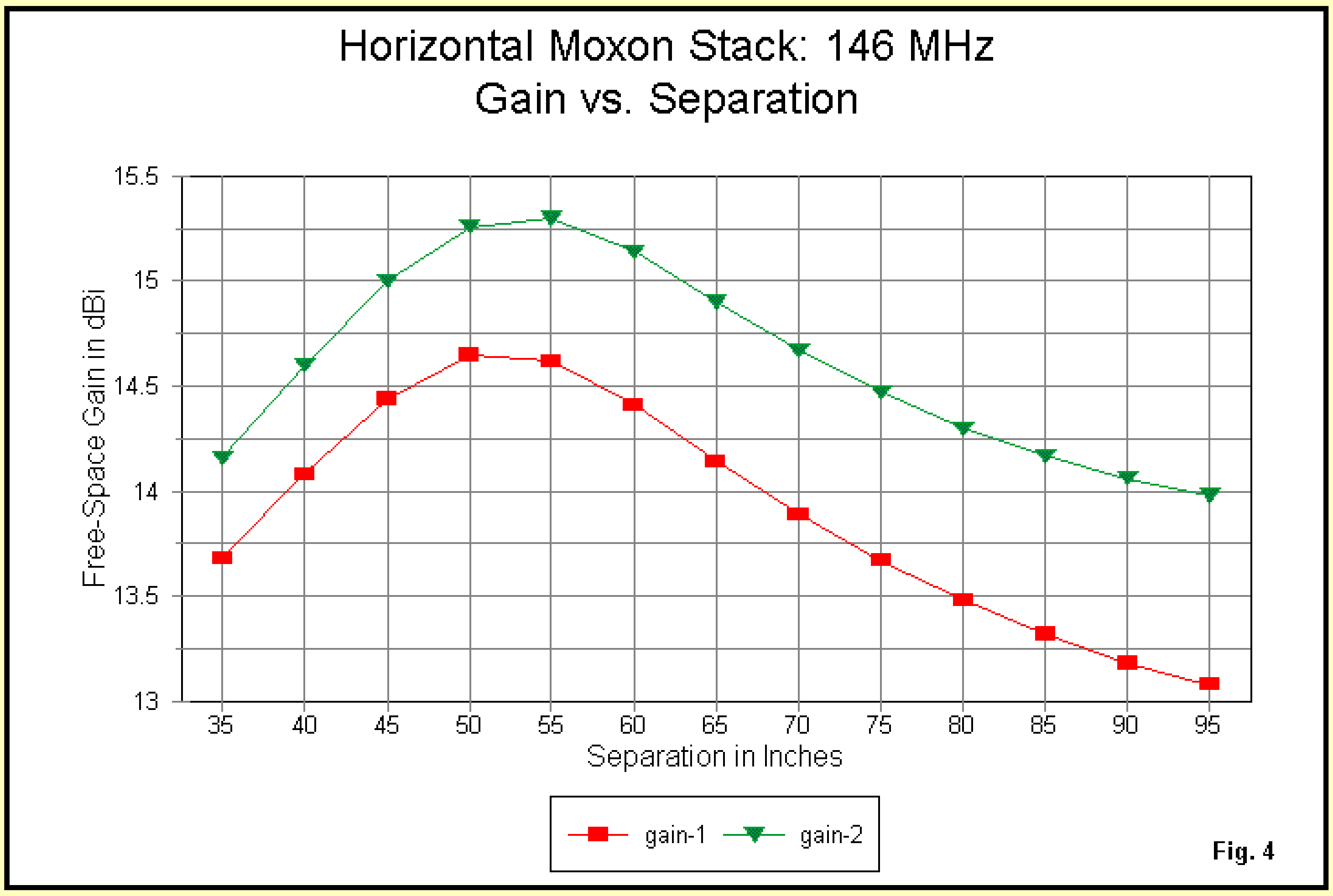

Fig. 4

Fig. 4 graphs the forward gain curves of the 2 array sets in order to note a special fact. The closer the array to the ground, the smaller the separation needed for achieving a maximum gain configuration. With a 2-wavelength base height, the array achieves maximum gain at about 1/16 wavelength greater separation between the antennas. Above about 2 wavelengths base height, the curves show an ever-decreasing differential so that the difference becomes virtually unnoticeable.

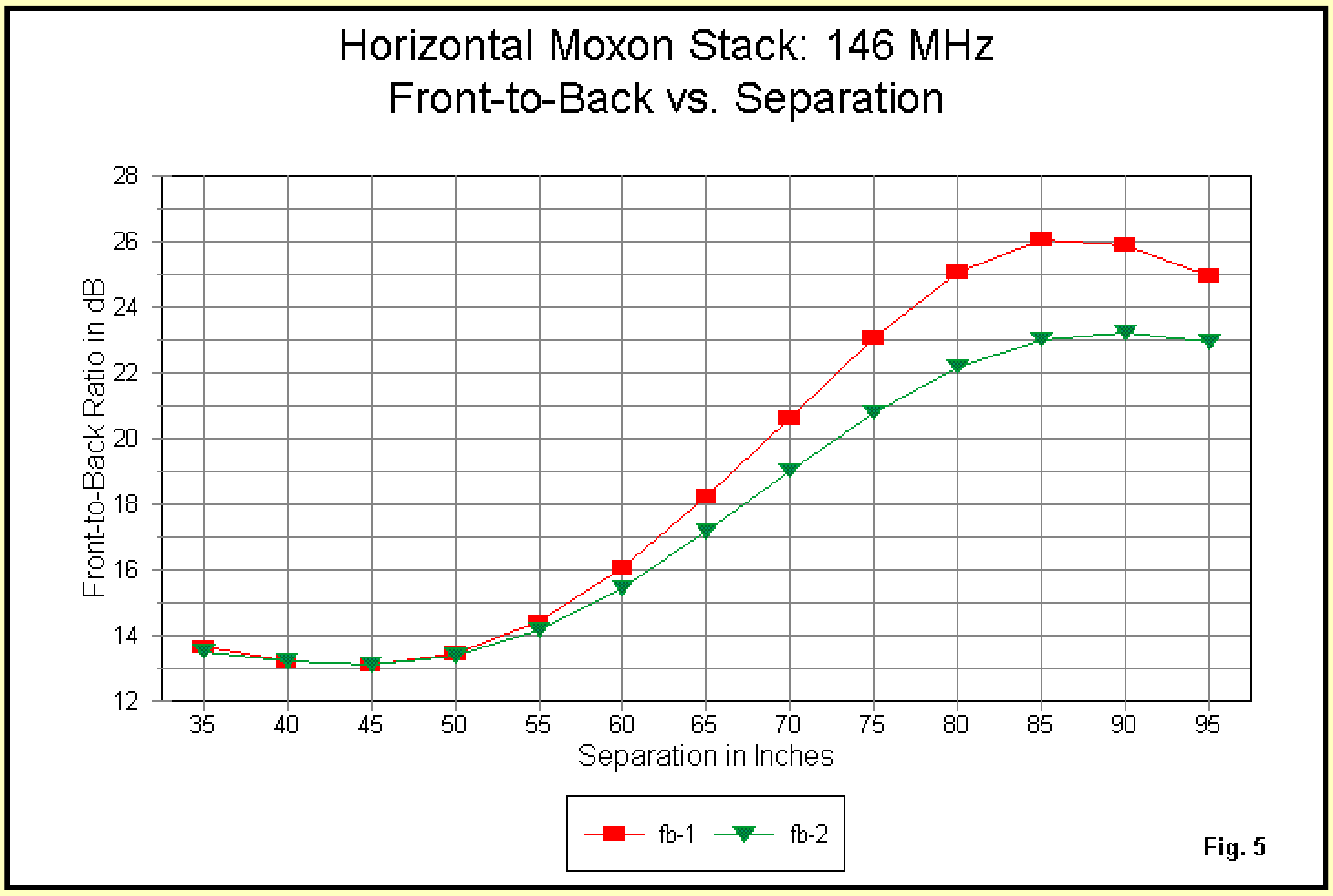

Fig. 5

Fig. 5 shows the front-to-back ratio of each stack with increasing separation. Once more, the stack with the greater base height requires slightly more separation to achieve its maximum.

There is one more interesting facet of the separation between arrays: the effect of separation upon the TO angle. For the VHF series of beams, I used a pattern increment of 0.1 degree to achieve relatively fine differentiations in TO angles, especially with the 1-wavelength base-height array. As shown in **Fig. 6**, the change in TO angle does not form a linear or other simple curve. With a separation close to 7/8 wavelength, the change in TO angle per increment of separation slows very noticeably, only to speed up again to a more normal rate of about 0.3 degrees per increment.

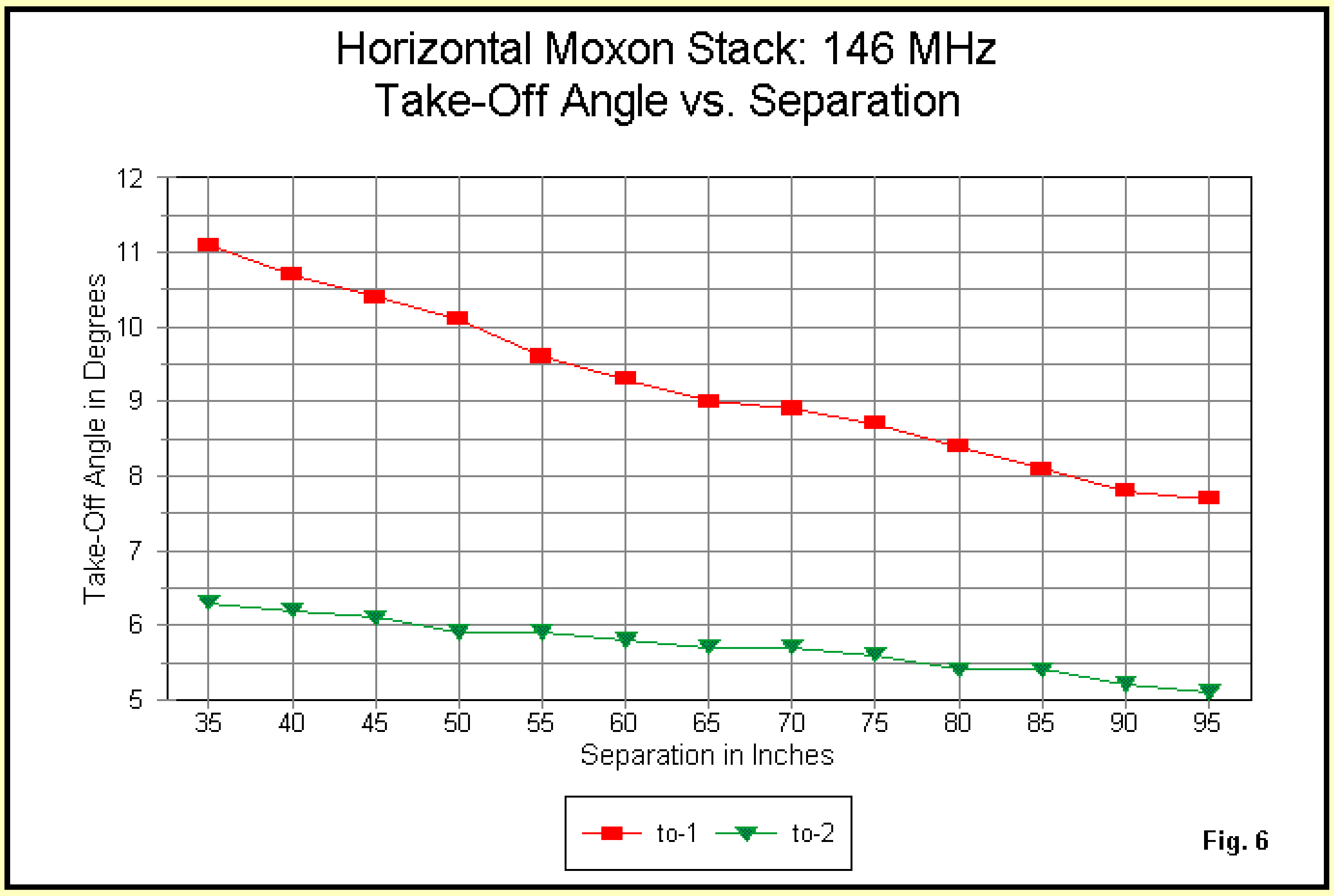

Fig. 6

The TO-angle exercise simply illustrates that fact that the interaction of arrays in a stack is far from a simple matter. You may survey the feedpoint data--comparing it to the values for a single antenna--for the various amounts of separation. Note that there is very little difference between the sets of each base height. However, within each set, both the resistance and reactance vary considerably when we stack Moxon rectangles. The maximum gain separation tends to yield feedpoint values that are close to maximally divergent from the single antenna. In fact, the feedpoint values most distant from those of a single antenna occur with a separation that marks the front-to-back minimum level.

Repeating the Experiment on 20 Meters

To ensure that the stacking phenomena that we observed at 2-meters are not unique to VHF stacks, I repeated the experiment. This time I used a frequency of 14.175 MHz, where 1 wavelength is just under 70'. I examined the data for baselines of 70' and 140', corresponding roughly to 1 and 2 wavelengths, respectively. The interval for the data scan is 5', with each increment being just over 0.07 wavelength. The 20-meter data appears in the following tables.

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20-Meter Moxon 2-Stack: 14.175 MHz: Base Height: 70' (Approx. 1 wl)

Single Antenna	Gain dBi	Front-Back Ratio dB	TO Angle degrees	Feed Z R+/-jX Ohms
	11.47	24.79	14	54 + j 6

Separation Ft.	WL	Gain dBi	Front-Back Ratio dB	TO Angle degrees	Top Z R+/-jX Ohms	Bottom Z R+/-jX Ohms
30	0.4324	13.78	13.33	11	62 + j 14	64 + j 13
35	0.5044	14.24	12.98 -	11	66 + j 10	67 + j 11
40	0.5765	14.63	13.13	10	68 + j 2	69 + j 5
45	0.6485	14.77 +	14.03	10	63 - j 6	66 - j 3
50	0.7206	14.61	15.80	10	54 - j 9	59 - j 6
55	0.7926	14.35	18.43	9	48 - j 6	53 - j 5
60	0.8647	14.07	21.44	9	46 - j 2	49 - j 2
65	0.9368	13.81	24.52	9	45 + j 1	47 + j 1
70	1.0088	13.60	26.53 +	8	46 + j 3	47 + j 4
75	1.0809	13.43	26.47	8	46 + j 5	48 + j 7
80	1.1529	13.25	24.90	8	46 + j 6	49 + j 9

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20-Meter Moxon 2-Stack: 14.175 MHz: Base Height: 140' (Approx. 2 wl)

Single Antenna	Gain dBi	Front-Back Ratio dB	TO Angle degrees	Feed Z R+/-jX Ohms
	11.79	28.49	7	52 + j 5

Separation Ft.	WL	Gain dBi	Front-Back Ratio dB	TO Angle degrees	Top Z R+/-jX Ohms	Bottom Z R+/-jX Ohms
30	0.4324	14.21	13.21	6	63 + j 14	66 + j 13
35	0.5044	14.74	13.00 -	6	66 + j 10	66 + j 10
40	0.5765	15.18	13.16	6	68 + j 3	69 + j 4
45	0.6485	15.39 +	13.93	6	64 - j 6	65 - j 5
50	0.7206	15.31	15.44	6	55 - j 9	58 - j 8
55	0.7926	15.06	17.51	6	48 - j 7	51 - j 7
60	0.8647	14.78	19.83	6	45 - j 3	47 - j 3
65	0.9368	14.53	21.93	6	44 + j 1	45 + j 0
70	1.0088	14.37	23.14	5	44 + j 3	45 + j 4
75	1.0809	14.23	23.28 +	5	44 + j 5	46 + j 6
80	1.1529	14.12	22.64	5	46 + j 7	47 + j 9

. . . .

Within the limits of correspondence between the increments (in terms of a wavelength) of separation between the 2 exercises, the 20-meter data very precisely parallels the 2-meter data. The maximum gain configuration requires a separation of about 0.65 wavelength. This value applies to both base heights, but the curves in **Fig. 7** show a tilt to the curve of the greater base height that corresponds to the same tilt in the 2-meter curves.

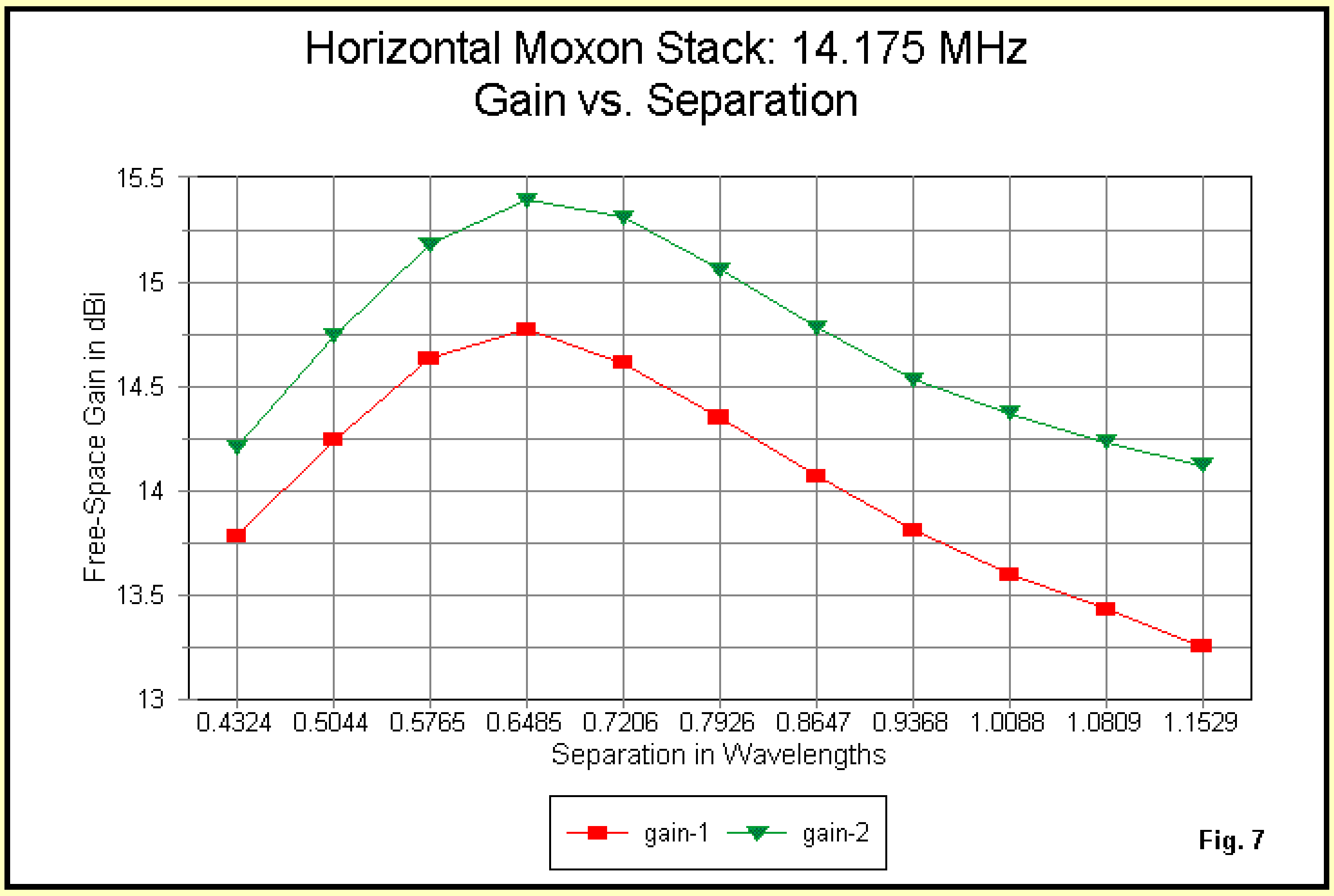

Fig. 7

Likewise, the maximum front-to-back configuration requires a slightly wider separation (about 1/16 wavelength) when the base height is 2 wavelengths relative to a base height of 1 wavelength. Moreover, the separation requirements are virtually identical for the 2-meter and the 20-meter stacks. **Fig. 8** shows the 20-meter front-to-back curves relative to the separation distance.

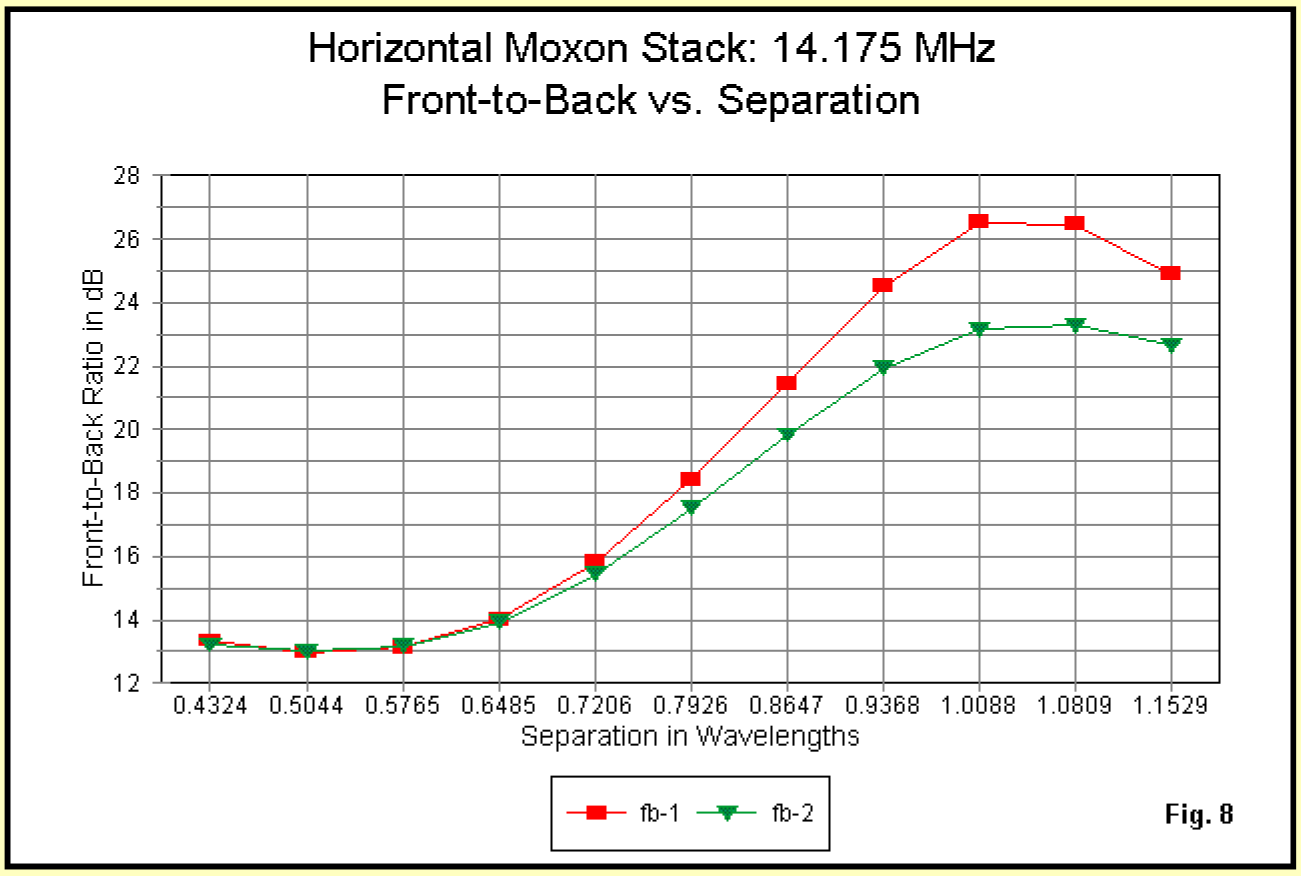

Fig. 8

Since the 20-meter exercise utilized pattern increments of 1.0 degree, the TO data is insufficiently refined to graph the rate of change. However, allowing for a slight difference in the feedpoint impedances of the 20-meter and the 2-meter single antennas, the feedpoint impedance data describe the same curves in both the resistance and the reactance columns.

Should We Stack Moxon Rectangles?

Achieving a maximum gain configuration for a vertical stack of horizontally oriented Moxon rectangles requires a moderate amount of separation--about 0.65 wavelength. However, the available front-to-back ratio of the stack is very low compared to the front-to-back ratio of a single antenna.

Maximizing the front-to-back ratio in the stack requires a far wider separation, something close to 1 wavelength. This separation is not likely to be feasible at HF in modest antenna installations, but may be possible in the VHF and upward regions. However, the cost of that achievement is 1 dB less gain than we can obtain from the maximum gain configuration. Indeed, the gain improvement over a single antenna at the base height is down to 2 dB.

The situation that applies to stacking Moxon rectangles applies to a considerable degree to standard Yagis using only 2 elements. to a lesser but significant extent, it also applies to larger Yagis up to 4-5 elements. For some data on Yagi stacks of 2 using various length arrays, see the notes titled "Supplementary Notes on Stacking" at ../stacksup.html.

A 3-stack of Moxons fares even worse. Besides the differential between the required separation for maximum gain and maximum front-to-back ratio, we encounter another problem. The very wide vertical (H-plane) beamwidth of the moxon results in extreme interaction between the arrays. The middle beam takes the brunt of the interaction. Its feedpoint impedance in a stack of three with the sources all in phase is almost 50% higher than the impedance of a single antenna and well-above the feedpoint impedances of the outer 2 antennas. Hence, equal power distribution to the antennas approaches a problematical level.

A Few General Notes on Stacking Horizontal Antennas

The interaction of any two antennas in a vertical stack is more complex than may appear at first sight. Perhaps the most common observation about the proper stacking height to achieve maximum gain concerns its relationship to the gain of an individual array in the stack. The higher the gain of the individual antenna in the stack, the farther apart the arrays must be to achieve maximum gain. For standard Yagis ranging from about 3 to 7 elements or so, we may even develop a rule of thumb:

$$S_\lambda \approx \frac{G_{dBi} - 2}{7.75} \tag{1}$$

where S(lambda) is the required separation in wavelengths and G(dBi) is the free-space gain of the individual antenna in the stack.

We know that this is a mere rule of thumb by virtue of the fact that we do not obtain a cancellation of units of measure. However, the quasi-equation does yield a first-order approximation of the required separation for maximum gain. However, it tends to go inaccurate when we deal with 2-element Yagis and other small arrays. There are reason for this--other than the rough-and-ready nature of the rule.

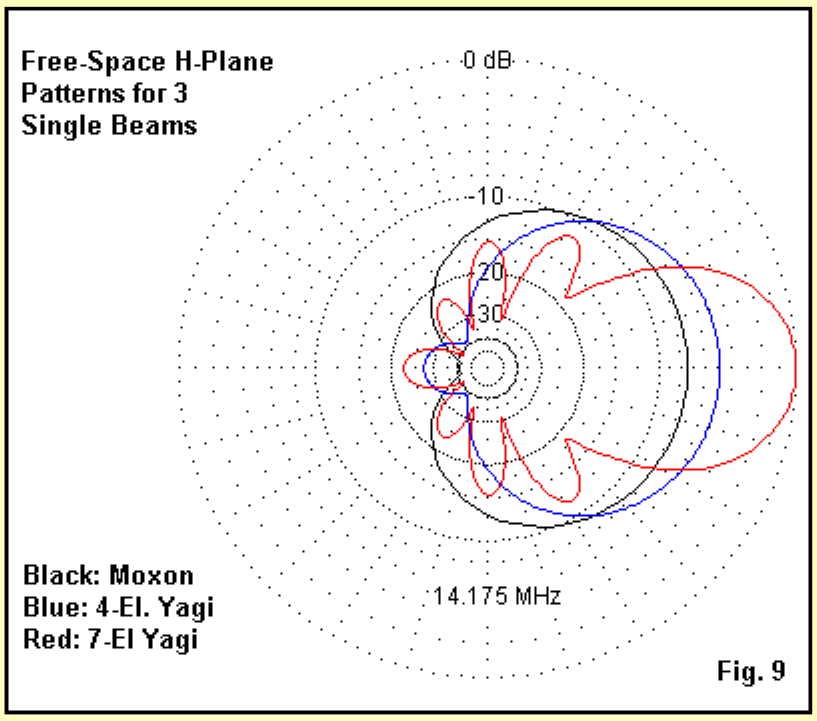

Fig. 9 shows the free-space E-plane patterns of 3 different arrays superimposed in a single plot. The longest array is a 7-element Yagi, which shows the narrow beamwidth associated with longer arrays. The 4-element Yagi shows a very wide beamwidth, but without any secondary lobes, such as those associated with the 7-element Yagi. The least strong pattern belongs to the 2-element Moxon rectangle. Although it has the lowest forward gain, it shows the greatest beamwidth at over 140 degrees between -3 dB points. As well a good bit of the side-energy lies behind the points that are 90 degrees each side of the main forward lobe bearing.

What the plots illustrate is that the interaction of two antennas in a stack is more dependent upon the shape and strength of the E-plane structure of the antenna's pattern than upon gain alone. In fact, we can begin by creating a rule of thumb for maximum gain configuration separation in terms of the antenna's vertical beamwidth.

$$S_\lambda \approx 2.66 + \log_{10}\left(\frac{1}{VBW - 27}\right) \tag{2}$$

where S(lambda) is the required separation between stacked antennas in wavelengths and VBW is the free-space vertical beamwidth of an individual antenna in the stack. This approximation is also only apt between about 3 and 7 elements. It may be useful for longer arrays but lacks testing for larger Yagis.

The approximation cannot be fully precise because it does not account for antenna radiation close to the 90-degree points of the array. The 4-element antenna E-plane shows radiation at the 90-degree points, but the 7-element Yagi has lower level sporadic lobes in these regions. If we change the strength of these lobes, then we also change the level of interaction between antennas and hence possibly change to a small degree the required separation for maximum gain from a stack.

The Moxon has radiation to the vertical side regions that extends smoothly well to the rear of the 90-degree points. So, too, do 2-element reflector-driver Yagis and similar small arrays. This phenomenon changes the nature of the interaction, widening the required spacing relative to the rules of thumb. Indeed, two dipoles in a stack present intersecting circles and require about a 5/8-wavelength spacing for maximum stack gain. The required separation figure remains in this region--0.6 to 0.65 wavelength--until we start working with long-boom 3 element Yagis.

The reason why we increase spacing with additional gain and its associated forward lobe shape is--in part--the greater distance between the antenna and the intersection of the forward lobes. **Fig. 10** provides some guidance here, with plots of 2-stacks of each of our antennas in **Fig. 9**, each stack set for maximum gain in free space.

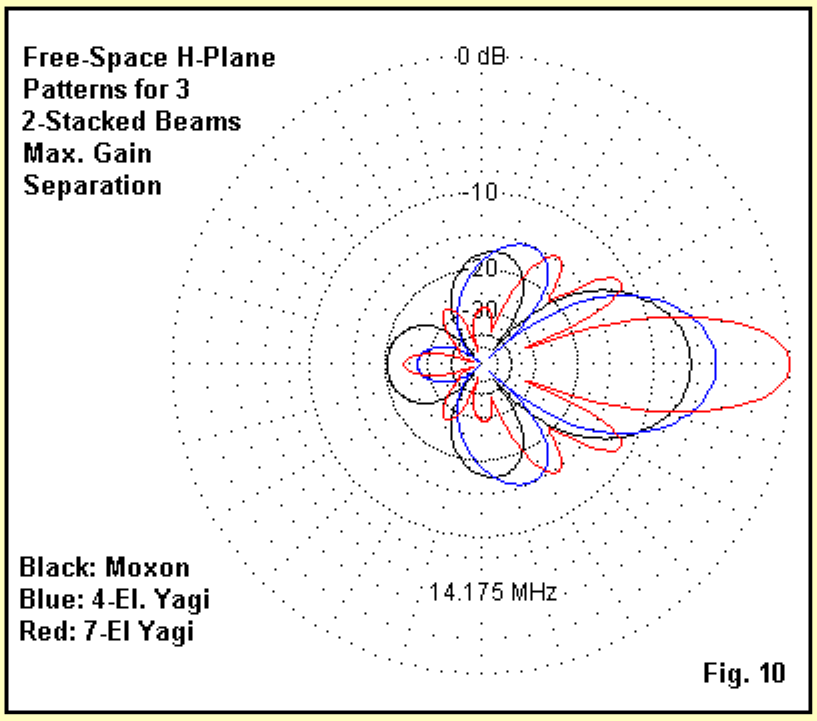

Fig. 10

The 7-element Yagi requires a spacing of about 1.45-1.48 wavelengths for maximum gain. Maximum gain in the forward lobe requires a convergence of the main forward energy from the individual antennas in the stack. The wide separation needed for maximum gain does not place the antennas in the right position for convergence of the strongest forward secondary lobes from each antenna. Hence, you will find an additional forward lobe compared to the single antenna pattern in **Fig. 9**. The rearward lobes are wide enough and weak enough to be unchanged in shape. However, the lobes at 90-degrees from the forward bearing are weakened by cancellation that occurs in the regions that are multiples of 1/2-wavelength separation. The antennas in the 7-element Yagi stack are nearly 1.5 wavelength apart.

Pattern convergence also appears with some clarity in the patterns for the 4-element Yagi stack--with a separation of 0.76 wavelength--and the Moxon stack--with a separation of 0.63 wavelength. Note that the 4-element Yagi, which had no sidelobes as a single antenna, now has major sidelobes in the E-plane. The pattern of addition and cancellation of radiation bends these sidelobes forward of the 90-degree points, which coincides with the fact that the single 4-element Yagi had most of its side-ward energy forward of the 90-degree points. However, the Moxon sidelobes are almost aligned with the 90-degree points, because so much of the side-ward energy was to the rear of the 90-degree points. In fact, the rearward energy from the individual antennas combines to give the stack a very considerable rear lobe, reducing the front-to-back ratio relative to the performance on an individual Moxon rectangle.

If we need a further reason why simple rules of thumb break down with certain individual arrays and with arrays beyond the limits of the rule, we need only consider that another set of radiation combinations occur when we stack antennas over real ground. At a distance from the antenna stack, reflected and direct radiation combine to yield patterns like those in **Fig. 2** and **Fig. 3**. The greater distance of the upper antenna from the ground than the lower antenna yields a slightly different phase angle to any pair of rays that we wish to combine to produce the strongest forward lobe. We have already seen that the elevation angle of the main forward lobe is roughly equal to that of a single antenna placed about 2/3 of the way upward between the actual antennas in the stack. Indeed, there will be small differences in the required separation for maximum gain as we move the base height from 1 wavelength to 2 wavelengths. The rules of thumb for maximum gain separation are most accurate applied to arrays with a base height of 2 wavelengths or more, even though the error for lower heights is small.

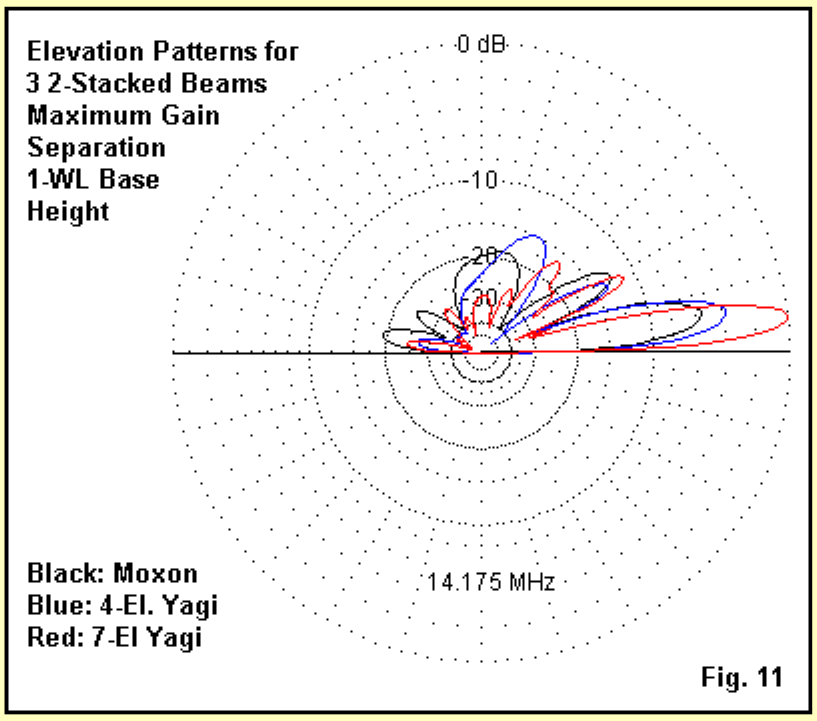

Fig. 11

Not only does the forward lobe receive some modification from ground reflections; so, too, do the remaining lobes. **Fig. 11** shows elevation patterns for our 3 stacks, each with a base height of 1 wavelength (about 70') above average ground. Notable are the additional lobes and nulls in the individual patterns that result from ground reflections combining with direct radiation. However, in general terms--and except for the elevated main forward and rearward lobes, the total lobe structure fits within the "shell" of the simpler free-space pattern. **Fig. 12** gives an example using the 7-element Yagi.

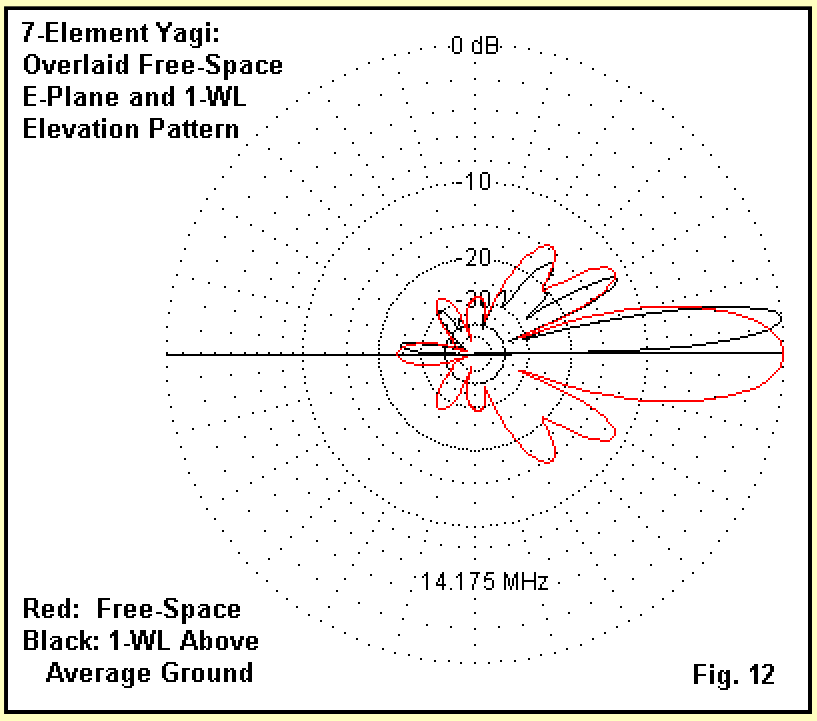

Fig. 12

Calculating the total effects of both the stacked-antenna interactions and the interaction of reflected and direct radiation might seem a daunting task. However, the calculations are routine within antenna modeling programs such as NEC, from which all of the patterns shown have emerged. Indeed, the surest way of finding the appropriate separation for two individual antennas in a stack is the use of such software. In this arena, modeling can turn rules of thumb into nimble fingers on the computerized abacus.

"Appropriate separation" does not necessarily mean the use of the maximum gain separation. If we recall the tables for the Moxon rectangles, we may achieve a front-to-back ratio of at least 20 dB at a separation of about 0.85 wavelengths, about halfway between the maximum gain and maximum front-to-back separations. At this height, rather than losing a full dB relative to maximum gain, we lose only about a half dB. In addition, the feedpoint resistance and reactance values are quite manageable for an in-phase feed system. Hence, for some operations, this compromise setting might constitute the optimal separation between rectangles in a stack. For other antennas, we can only find comparable appropriate separations by careful analysis in small separation increments. Once more, the swift calculations of modeling software earn their keep.

Our voyage through the world of stacked arrays, with attention to the Moxon rectangle, is only half a story. We can also stack the rectangles when they are positioned vertically--as we might wish to do for improved gain for VHF/UHF FM service. That is a story for another episode.

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