Some Notes on Two-Element Horizontal Phased Arrays
Part 1: The Limits of Performance

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The directional 2-element phased array achieved notoriety in the 1950s with builder claims that one or another variation on the basic design outperformed 3- and even 4-element Yagis. Although we now know that the appearance of high performance owed much to Yagi deficiencies of the period, horizontal phased arrays have retained much of their mid-century aura of magic. Since magic and an understanding of antennas are mutually exclusive, perhaps we should begin again.

The notes in this series will begin with some basic modeling data that tends to set limits to the performance expectations that we may logically have of 2-element phased arrays. In the second part, we shall explore the degree to which the geometry of the parasitic array can capture the potential of phased element performance. Part 3 will examine one of the two classic methods of array phasing: the ZL-Special with its single phase line. In Part 4, we shall look at two different ways of phasing a pair of elements using element-matching techniques, one by R. Baumgartner, HB9CV, the other by Eric Gustafson, N7CL. Throughout. we shall try to integrate specific design strategies into an overall picture of the performance of which 2-element phased arrays are capable.

A Few Preliminaries

The idea of a 2-element phased array contains an ambiguity. At the most general level, the notion can refer to the relative phasing of the elements in any 2-element array. Under this heading, we may include arrays with a single driven element as well as two driven elements. The perspective offered by this most general idea of a phased array will be useful in seeing where some antennas fit into a larger picture.

Alternatively, the concept of a 2-element phased array often refers specifically to an "all-driven" antenna, that is, to an array in which both elements receive power directly from the source. The key question that immediately arises within this view of phased arrays is how we may get energy to the individual elements in the correct magnitude and phase to effect a desired set of performance characteristics. The most common means is via a "phasing line" composed of a length or lengths of transmission line. Indeed, this means of conveying energy from the array source to the individual elements has been the source of numerous misconceptions about how phased arrays operate.

The phasing-line system of energy transfer, of course, is quite unnecessary. As Brian Egan, ZL1LE, demonstrated with a 15-meter phased array in the 1990s, one may create a phasing network of lumped components and then use separate lines to each element so long as they preserve the relative values of current magnitude and phase created by the network.

The key to understanding 2-element horizontal phased arrays is the fact--stressed by Roy Lewallen, W7EL, in many writings--that the relative current magnitude and phase between the two elements determines the operating characteristics of the antenna. In the early days of phased-array popularity, most builders thought in terms of the impedance transformation along a transmission line linking the elements. However, the impedance along a mismatched line does not track with the current magnitude and phase transformations along the line. Impedance values repeat on a lossless line twice for each wavelength of line. However, current magnitude and phased values appear only once per wavelength.

From this misunderstanding others emerged. Although the most popular line lengths interconnecting elements were in the vicinity of 1/8 wavelength, most folks thought in terms of a 135-degree phase shift. However, with or without a half twist in the short line, the current can only make an approximate 45-degree phase shift. (The number is a crude marker, since we have already noted that the current phase may change more or less than 45-degree in a line that is 45-degree long.) If a straight line yields a 45-degree phase shift in current, then a line with a half twist yields a -45-degree phase shift. Antenna patterns may be identical to those produced by feeding the elements 135 degrees out of phase, but the current behavior and the consequences for evaluating means of obtaining the correct phasing of the elements will depend upon the -45-degree perspective. Because we shall be looking at close-spaced element systems, we shall adopt this orientation throughout these notes.

A further constraint upon our understanding of 2-element horizontal arrays has been the magic associated with 1/8-wavelength spacing. In fact, no particular spacing between elements holds any theoretically superior place in the scheme of 2-element arrays. We shall discover that in some respects, almost any spacing will do, although specific spacings between elements can result in arrays that are easier to implement.

A Modeling Project

In a number of past articles, I have presented a partial portrait of phased array performance potentials, for example, in the series in NCJ on log-cell Yagis. In the remainder of this first set of notes, I want to expand our appreciation of phased array performance parameters, although space will not allow an absolutely complete account.

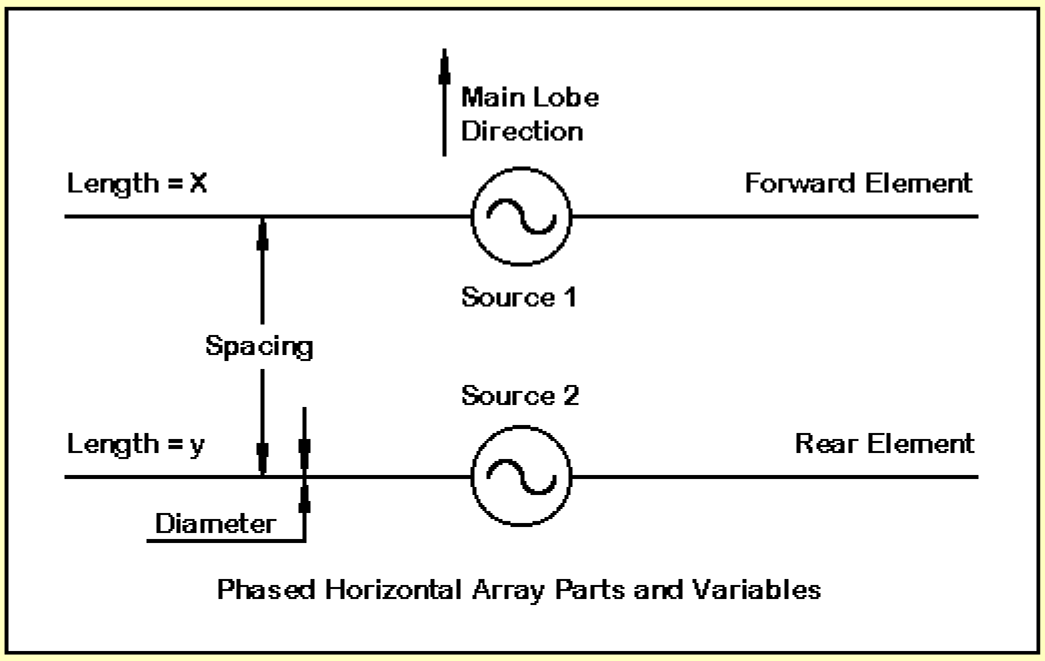

Fig.1-1 The basic parts and structural variables of a 2-element horizontal phased array.

Fig. 1-1 presents the basic parts of a 2-element phased array as we shall model it in NEC-4. We shall assign to each element a current source, specifying both the magnitude and phase angle. By convention, the designated forward element will have a current magnitude of 1.0 and a phase angle of 0.0 degrees. The designated rear element will then be assigned the values of current magnitude and phase that yield a desired performance limit. Since we are working with directional arrays with a single main forward lobe, the forward element will always be the element in the direction of that lobe. Assigning separate values of current magnitude and phase angle to each element is an analog of what we accomplish with a phasing network. Such networks cannot yield performance that exceeds the limits of separate sources for each element, no matter the ingenuity of the system.

For the notes in this section, we shall reduce the total number of variables to a manageable number. We shall vary the spacing between elements systematically. We shall also examine some variations in element length, using both equal-length and unequal-length elements in the study. However, these results will change if we alter the diameter of the elements. For convenience, we shall employ 10-meter (28.5-MHz) elements made from 0.5" diameter aluminum. These elements give us a reasonably realistic model that scales easily to other amateur bands. With a fixed element diameter, we shall not explore variations that result from selecting other diameter materials.

When exploring sources with a relative phase angle between 0 and -90-degree, we must simulate the line half twist by setting up the model elements in opposite directions. That is, if the forward elements extends from a - value to a + value, then the rear element extends from a + value to a minus value. Adhering to this modeling scheme keeps the instantaneous current directions correct.

The basic element for our exploration is a resonant dipole of the specified material. In a NEC-4 model, such a dipole is 197.6" long or about 0.4771 wavelength long at 28.5 MHz. (The half-inch diameter element is 0.001207 wavelength across.) The subject dipole has a resonant impedance of 72.1 + j 0.5 Ohms. Now we are finally ready to examine a 2-element phased array.

Maximum Front-to-Back Ratio Configurations

The basic model consisted of two self-resonant dipoles of the type just described set at various distances apart. The spacing ranged from 0.05 wavelength to 0.2 wavelength in 0.025-wavelength increments. This range covers--with some interesting but practically useless excess--the element spacing used in virtually all recorded directional phased array construction.

In addition to using equal-length self-resonant elements, I also made up pairs that are 10% shorter and 10% longer than the basic model. The short elements are 177.84" long (0.4294 wavelength), while the long elements are 217.36" long (0.5249 wavelength). As we shall see, resonance is not a requisite for a phased pair of elements. (We shall look at unequal-length elements soon.)

The first exercise attempted to arrive at the rear element relative current magnitude and phase angle necessary top achieve a maximum 180-degree front to back ratio. Although the pursuit of a perfect null can go on indefinitely, it proved fairly easy to obtain a rear null greater than 60 dB lower than the forward lobe maximum value. Since the maximum null is a very narrow-bandwidth phenomenon, -60 dB seemed deep enough to show general trends when we set 2-element phased arrays for a maximum front-to-back ratio.

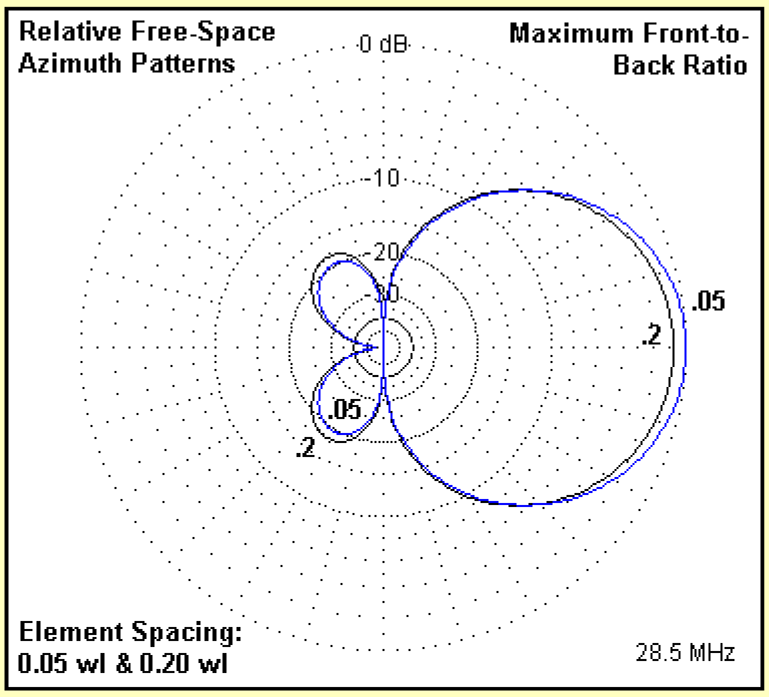

Fig. 1-2 Comparative free-space azimuth patterns of a 2-element horizontal phased array configured for maximum 180-degree front-to-back ratio with close-spaced and wide-spaced elements.

Fig. 1-2 shows typical patterns for the narrowest element spacing and the widest element spacing used. Although only one set of patterns appear in the figure, the general properties apply to all three of the subject models. As element spacing increases beyond 0.1 wavelength, gain drops off. More notable are the rear lobes. The deep null occurs within a rearward lobe, leaving angle side lobes. The lobes are weakest at the most narrow spacing levels and increase with wide spacing. To some degree, then, aiming at the maximum 180-degree front-to-back ratio may be operationally misdirected, although it serves to set operational limits for the 2-element array.

Equal-Length 2-Element Phased Array Performance
Maximum 180-Degree Front-to-Back Configuration

Model SHT-E **Element Length (Front and Rear): 0.2147 wl**
Frequency: 28.5 MHz **Diameter: 0.001207 wavelength (0.5")**

Space wl	Gain dBi	Front-to-Back Ratio dB	Z1 (Rear) R +/- jX Ohms	Z2 (Foprward) R +/- jX Ohms	Rear I Magnitude	Rear I Phase
0.05	**6.41**	**65.58**	**3.9 - j110.3**	**3.2 - j 78.8**	**1.024**	**-17.4**
0.075	**6.42**	**66.13**	**7.9 - j117.1**	**7.5 - j 71.8**	**1.035**	**-26.4**
0.1	**6.36**	**73.68**	**11.9 - j120.7**	**14.7 - j 64.0**	**1.045**	**-35.6**
0.125	**6.25**	**61.40**	**15.7 - j122.8**	**24.4 - j 57.6**	**1.051**	**-44.9**
0.15	**6.11**	**65.41**	**19.1 - j123.8**	**35.7 - j 53.7**	**1.056**	**-54.3**
0.175	**5.92**	**68.86**	**22.3 - j124.3**	**47.7 - j 53.0**	**1.057**	**-63.8**
0.2	**5.69**	**65.46**	**25.7 - j124.2**	**59.1 - j 55.8**	**1.057**	**-73.2**

Model RES-E **Element Length (Front and Rear): 0.2386 wl**
Frequency: 28.5 MHz **Diameter: 0.001207 wavelength (0.5")**

Space wl	Gain dBi	Front-to-Back Ratio dB	Z1 (Rear) R +/- jX Ohms	Z2 (Forward) R +/- jX Ohms	Rear I Magnitude	Rear I Phase
0.05	6.50	63.70	10.7 - j 35.5	-1.6 + j 7.0	1.033	-17.0
0.075	6.50	88.97	15.9 - j 37.5	4.6 + j 24.2	1.049	-26.0
0.1	6.44	60.36	20.7 - j 38.4	15.3 + j 39.4	1.063	-35.2
0.125	6.33	64.42	25.1 - j 38.8	29.8 + j 51.2	1.074	-44.7
0.15	6.18	66.73	29.1 - j 38.6	46.8 + j 58.2	1.080	-54.3
0.175	5.99	61.47	32.8 - j 38.2	64.6 + j 60.1	1.083	-64.0
0.2	5.76	63.34	36.0 - j 37.6	81.3 + j 56.7	1.080	-73.6

Model LNG-E **Element Length (Front and Rear): 0.2624 wl**
Frequency: 28.5 MHz **Diameter: 0.001207 wavelength (0.5")**

Space wl	Gain dBi	Front-to-Back Ratio dB	Z1 (Rear) R +/- jX Ohms	Z2 (Forward) R +/- jX Ohms	Rear I Magitude	Rear I Phase
0.05	6.59	66.90	18.9 + j 39.0	- 7.3 + j 95 5	1.045	-16.6
0.075	6.59	74.73	25.9 + j 41.8	1.0 + j125.3	1.067	-25.5
0.1	6.52	63.87	32.0 + j 43.8	16.4 + j150.6	1.087	-34.9
0.125	6.41	65.07	37.4 + j 45.5	37.7 + j169.4	1.101	-44.5
0.15	6.26	66.57	42.3 + j 47.0	62.8 + j180.7	1.110	-54.4
0.175	6.08	72.84	46.7 + j 48.6	88.8 + j183.6	1.113	-64.3
0.2	5.85	67.24	50.7 + j 50.0	113.3 + j178.5	1.110	-74.3

Note: All gain values are for free-space. Rear current (I) magnitude and phase values are relative to forward element values of 1.0 and 0.0 degrees. Model RES-E uses elements of equal length to an independent resonant dipole at the test frequency. Models SHT-E and LNG-E uses elements that are 10% shorter and 10% longer, respectively.

Table 1. Performance and operating conditions of 3 equal-length element 2-element phased arrays in a maximum 180-degree front-to-back ratio configuration.

Table 1 provides full data for the short, resonant, and long element pairs. As we might expect, the maximum gain for any spacing is partly dependent upon the element lengths. consistent among the three test models is the occurrence of maximum gain at the closest spacing levels: 0.05 and 0.075 wavelength. Thereafter, gain decreases steadily. The front-to-back values are simply for the record to verify that the model obtained the requisite depth of rear null.

At a spacing of 0.125 wavelength, a popular element separation for 2-element Yagis and phased arrays, the forward gain of the maximum-null phased arrays do not differ significantly from the gain of a well-designed Yagi. In the maximum front-to-back configuration, then, the phased array's claim to fame is only its rearward null and not its gain.

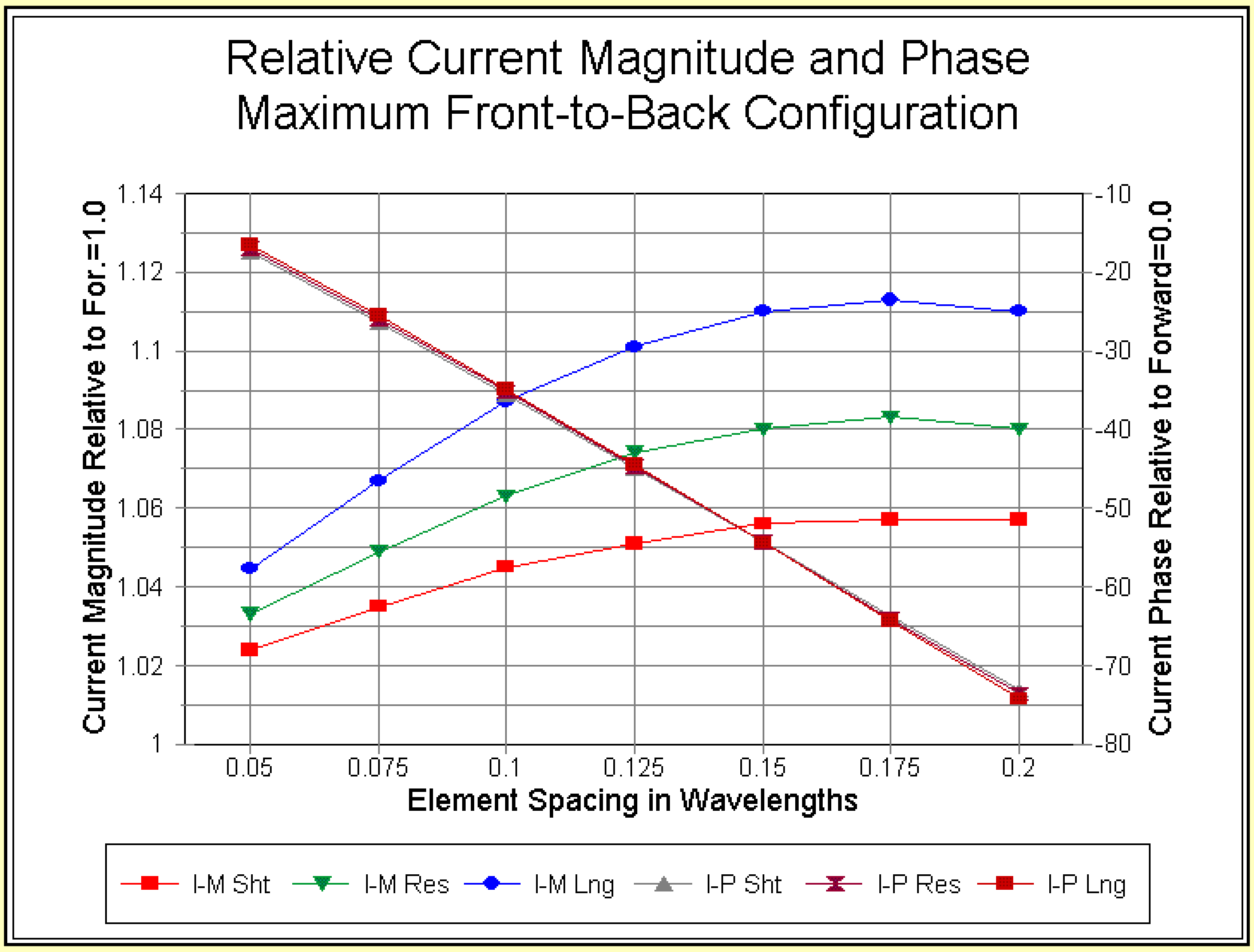

Fig. 1-3 The rear element relative current magnitude and phase angle for short, resonant, and long element lengths in arrays having equal- length forward and rear elements, and set for maximum front-to-back ratio. "I-M" means rear element relative current magnitude. "I-P" means rear element relative current phase. "Sht" refers to model SHT-E; "Res" refers to model RES-E; and "Lng" refers to model LNG-E. See Table 1 for model specifications.

Of primary interest to us are the rear element relative values of current magnitude and phase angle necessary to yield the deep null. **Fig. 3** graphically portrays the tabulated data of **Table 1**. Of immediate notice is that the change in element lengths between models has almost no effect on the requisite phase angles. The graphs of the three lines overlap and proceed in a virtually linear curve from about -17 degrees at 0.05-wavelength spacing to about -73 degrees at 0.2-wavelength spacing. Equally notable is the fact that we may obtain a rearward null for any spacing in this range.

What does change with the length of the elements is the relative current magnitude required on the rear element. The longer the element pair, the higher the required value of relative rear element current to achieve. The differentials for 10% changes in element length are between 2% and 3%.

Not all element spacings will be easy to implement with standard means of element phasing. The tabulated data shows negative resistance values in some entries for very close-spaced elements. These values are correct and simply mean that the mutual coupling between elements is providing more energy to the affected element than the source itself.

A Test of Equal vs. Unequal Element Lengths

There are three possible element arrangements for a 2-element horizontal phased array. As we have just examined, both elements may be equal in length. However, **Fig. 1-4** shows two more configurations. The forward elements may be shorter than the rear element, and the forward element may be longer than the rear element. Our familiarity with the requirements for parasitic beams makes one of the arrangements natural and the other almost unthinkable.

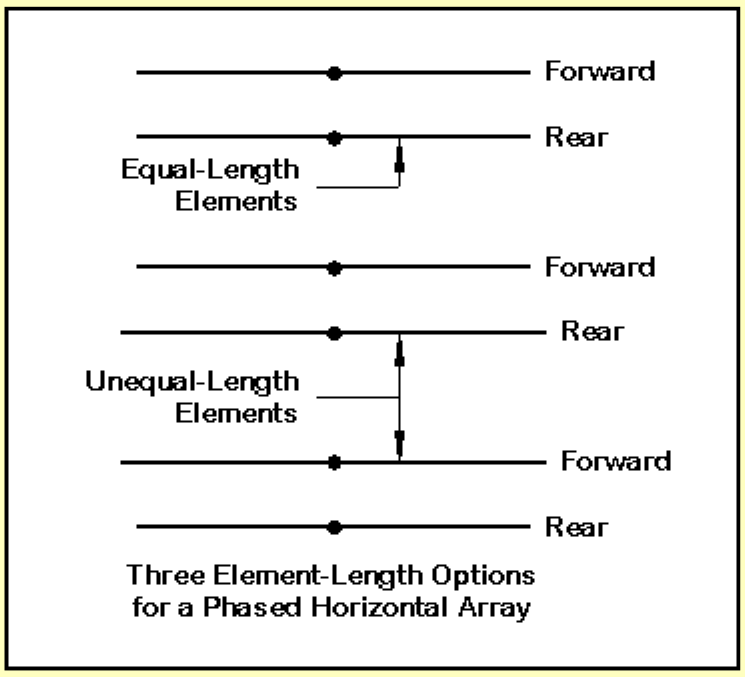

Fig. 1-4 Three options for element length relationships between the forward and rear elements of a 2-element phased array.

Nevertheless, both types of unequal-length element arrays are fully functional in a phased array. all that we need to do is provide the two elements with the correct relative current magnitudes and phase angles. **Table 2** provides the complete modeling data on the test runs. The equal-length model is the same as used for the earlier runs. Each of the unequal-length arrays has one element that is the same as our original self-resonant dipole and a second element that is 5% longer: 207.48" or 0.5010 wavelength. As the table shows, there is no significant difference in the maximum forward free-space gain. Once more, at the closest element spacing modeled, a negative resistive component on the forward element is possible.

Unequal-Length 2-Element Phased Array Performance
Maximum 180-Degree Front-to-Back Configuration

Model RES-UF **Element Length: Front: 0.2505 wl; Rear: 0.2386 wl**
Frequency: 28.5 MHz **Diameter: 0.001207 wl (0.5")**

Space wl	Gain dBi	Front-to-Back Ratio dB	Z1 (Rear) R +/- jX Ohms	Z2 (Forward) R +/- jX Ohms	Rear I Magnitude	Rear I Phase
0.05	6.52	62.19	12.1 - j 38.7	-3.6 + j 53.5	1.114	-16.9
0.075	6.52	69.26	17.3 - j 39.5	3.5 + j 75.7	1.139	-25.9
0.1	6.45	63.68	22.0 - j 39.7	16.4 + j 95.2	1.160	-35.3
0.125	6.34	65.22	26.2 - j 39.5	34.0 + j109.8	1.175	-44.9
0.15	6.20	63.14	30.1 - j 39.0	54.6 + j118.6	1.185	-54.6
0.175	6.02	67.19	33.5 - j 38.4	76.1 + j120.6	1.186	-64.4
0.2	5.79	68.90	36.6 - j 37.6	96.3 + j116.4	1.184	-74.2

Model RES-UR **Element Length: Front: 0.2386 wl; Rear: 0.2505 wl**
Frequency: 28.5 MHz **Diameter: 0.001207 wl (0.5")**

Space wl	Gain dBi	Front-to-Back Ratio dB	Z1 (Rear) R +/- jX Ohms	Z2 (Forward) R +/- jX Ohms	Rear I Magnitude	Rear I Phase
0.05	6.52	63.77	13.3 + j 4.5	-2.4 + j 3.7	0.962	-16.9
0.075	6.52	61.79	19.3 + j 3.7	3.7 + j 22.1	0.974	-25.9
0.1	6.45	67.19	24.8 + j 3.6	14.5 + j 38.0	0.985	-35.1
0.125	6.35	62.79	29.7 + j 3.7	29.2 + j 50.1	0.992	-44.5
0.15	6.20	65.18	34.2 + j 4.2	46.2 + j 57.4	0.997	-54.1
0.175	6.01	65.28	38.4 + j 4.9	64.0 + j 59.3	0.998	-63.7
0.2	5.78	63.36	42.1 + j 5.9	80.8 + j 56.3	0.998	-73.3

For comparative data on Model RES-E, see Table 1.

Note: All gain values are for free-space. Rear current (I) magnitude and phase values are relative to forward element values of 1.0 and 0.0 degrees. Model RES-E uses elements of equal length to an independent resonant dipole at the test frequency. Models RES-UF and RES-UR uses elements that are 5% longer than those in RES-E at the forward and at the rear elements, respectively.

Table 2. Performance and operating conditions of 2 unequal-length element 2-element phased arrays in a maximum 180-degree front-to-back ratio configuration.

Fig. 1-5 shows the relative current magnitude on the rear element, along with the relative phase angle. As with the three equal-element-length arrays, the phase angles required to achieve a 180-degree front-to-back ratio in excess of 60 dB overlap with considerable precision. The differences are almost solely in the realm of the required relative current magnitude for the rear element. In this figure and in **Fig. 1-3**, you will note a slight decrease in the rear element current magnitude at the maximum spacing used (0.2 wavelength). The reversal of direction in current magnitude is consistent for all models in the series, both the ones used here and others in my collection.

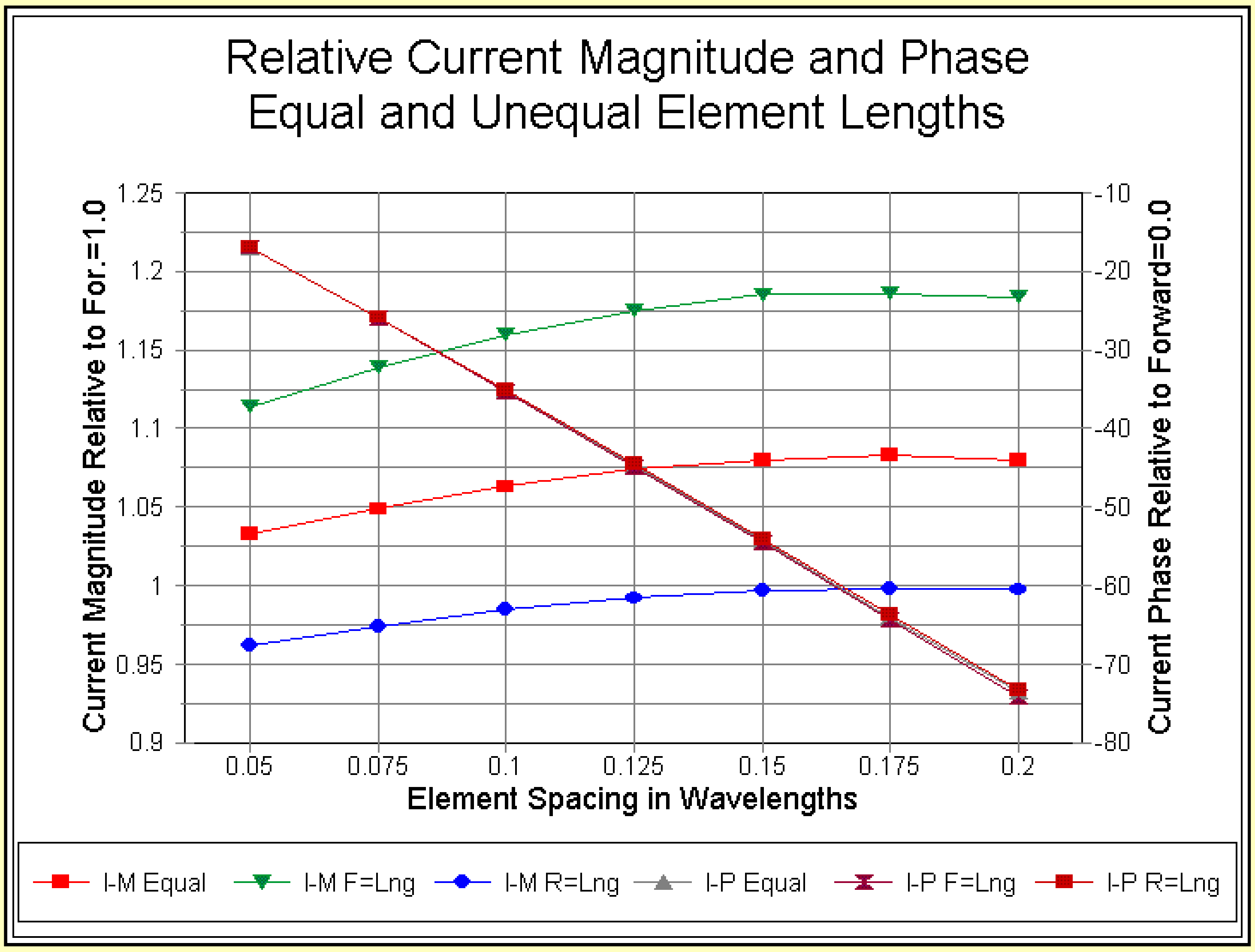

Fig. 1-5 The rear element relative current magnitude and phase angle for various element-length relationships in 2-element horizontal phased arrays that are set for maximum front-to-back ratio. "Equal" refers to model RES-E; "F=Lng" refers to model RES-UF; and "R=Lng" refers to model RES-UR, according to whether the elements are equal in length, the forward element is 5% longer, or the rear element is 5% longer. See Table 2 for the specifications of models RES-UF and RES-UR.

These models cannot guarantee that any particular element arrangement will provide an adequate basis for a practical array. However, when experimenting with phased arrays and various phasing schemes, it pays not to overlook the potential of a longer forward element.

Maximum Gain Configurations

The maximum front-to-back ratio configuration of a phased array represents one limit of performance, one marked by moderate gain and a deep rearward null. We may also set the relative current magnitudes and phase angles to achieve maximum forward gain, letting the front-to-back ratio become whatever it will be. In general, the conditions for maximum forward gain in a 2-element horizontal phased array do not favor high front-to-back ratios. **Fig. 1-6** shows a typical maximum gain pattern, with a front-to-back ratio well below 10 dB.

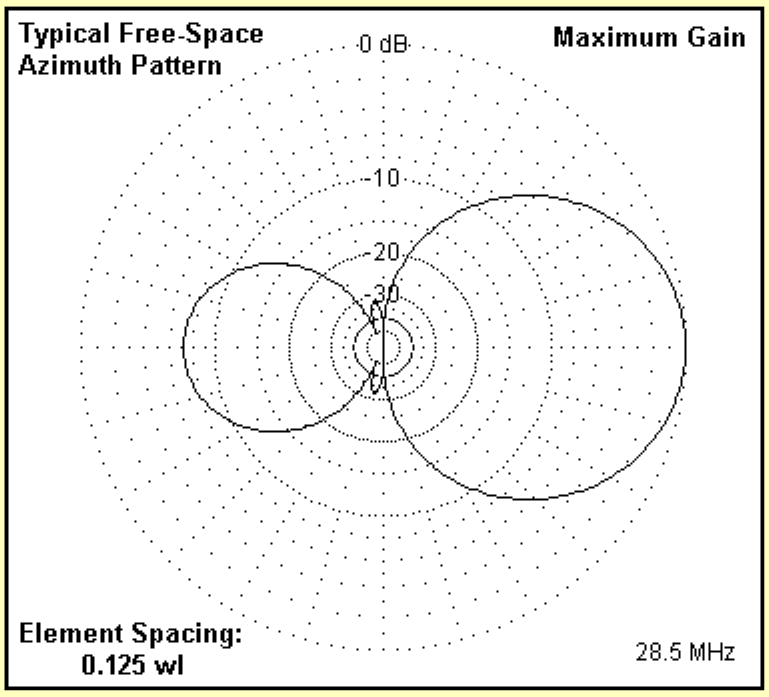

Fig. 1-6 A typical free-space azimuth pattern for a 2-element phased array set for maximum forward gain.

For the 5 models that we previously examined, **Table 3** provides the necessary data. Maximum gain does not occur at the very closest spacing tested, but appears in the 0.75 to 0.1 wavelength region of element spacing. Front-to-back ratios show a steady decrease with increasing element spacing. The maximum gain phenomenon has a wider bandwidth than the maximum front-to-back null. Therefore, each registered data set comprises a centered set of values in the middle of the range or phase angles and the range of current magnitudes that yield the highest gain. Over this region, the front-to-back ratio may change by as much as 2 dB, and the table shows only the center value.

Equal-Length and Unequal-Length 2-Element Phased Array Performance
Maximum Gain Configuration

Model SHT-E **Element Length (Front and Rear): 0.2147 wl**

Frequency: 28.5 MHz Diameter: 0.001207 wl (0.5")

Space wl	Gain dBi	Front-to-Back Ratio dB	Z1 (Rear) R +/- jX Ohms	Z2 (Forward) R +/- jX Ohms	Rear I Magnitude	Rear I Phase
0.05	7.10	8.57	1.8 - j102.1	1.5 - j 87.4	1.010	- 8.0
0.075	7.23	7.57	3.5 - j104.5	3.2 - j 85.1	1.015	-11.0
0.1	7.24	7.42	5.4 - j105.0	6.2 - j 80.1	1.020	-14.8
0.125	7.21	7.01	7.0 - j103.8	10.5 - j 75.5	1.020	-18.0
0.15	7.15	6.67	8.8 - j101.9	15.5 - j 70.4	1.025	-21.3
0.175	7.06	6.33	10.6 - j 99.7	21.4 - j 66.5	1.025	-24.5
0.2	6.96	5.95	12.6 - j 97.2	27.7 - j 63.4	1.030	-27.5

Model RES-E Element Length (Front and Rear): 0.2386 wl
Frequency: 28.5 MHz Diameter: 0.001207 wl (0.5")

Space wl	Gain dBi	Front-to-Back Ratio dB	Z1 (Rear) R +/- jX Ohms	Z2 (Forward) R +/- jX Ohms	Rear I Magnitude	Rear I Phase
0.05	7.19	8.17	4.9 - j 24.6	-0.7 - j 5.7	1.015	- 7.5
0.075	7.31	7.72	7.0 - j 21.8	2.0 + j 5.1	1.020	-11.0
0.1	7.31	7.38	9.5 - j 18.1	6.2 + j 15.9	1.030	-14.5
0.125	7.28	7.08	11.8 - j 14.2	12.3 + j 25.5	1.038	-18.0
0.15	7.21	6.75	13.7 - j 10.2	20.3 + j 33.7	1.035	-21.5
0.175	7.13	6.46	16.1 - j 6.3	28.9 + j 40.6	1.040	-25.0
0.2	7.02	5.97	18.3 - j 1.7	37.9 + j 45.1	1.040	-27.8

Model LNG-E Element Length (Front and Rear): 0.2624 wl
Frequency: 28.5 MHz Diameter: 0.001207 wl (0.5")

Space wl	Gain dBi	Front-to-Back Ratio dB	Z1 (Rear) R +/- jX Ohms	Z2 (Forward) R +/- jX Ohms	Rear I Magnitude	Rear I Phase
0.05	7.28	8.48	9.3 + j 52.5	- 3.8 + j 78 0	1.025	- 7.5
0.075	7.39	7.71	11.8 + j 62.1	0.3 + j 98.1	1.030	-10.8
0.1	7.39	7.47	14.5 + j 69.9	7.0 + j116.4	1.035	-14.5
0.125	7.35	7.11	17.5 + j 77.5	15.6 + j132.1	1.045	-18.0
0.15	7.29	6.77	20.3 + j 84.5	26.6 + j145.2	1.050	-21.5
0.175	7.20	6.43	23.4 + j 91.1	39.2 + j155.7	1.055	-25.0
0.2	7.10	6.03	26.1 + j 97.6	53.0 + j162.6	1.050	-28.3

Model RES-UF Element Length: Front: 0.2505 wl; Rear: 0.2386 wl
Frequency: 28.5 MHz Diameter: 0.001207 wl (0.5")

Space wl	Gain dBi	Front-to-Back Ratio dB	Z1 (Rear) R +/- jX Ohms	Z2 (Forward) R +/- jX Ohms	Rear I Magnitude	Rear I Phase
0.05	7.21	8.19	5.2 - j 28.3	-1.1 + j 38.7	1.085	- 7.5
0.075	7.33	7.74	7.6 - j 24.2	1.7 + j 53.1	1.100	-11.0
0.1	7.33	7.36	9.7 - j 19.8	7.1 + j 66.5	1.110	-14.5
0.125	7.30	7.04	11.9 - j 15.3	14.3 + j 78.6	1.120	-18.0
0.15	7.23	6.70	13.7 - j 11.0	23.7 + j 88.6	1.120	-21.5
0.175	7.15	6.40	16.0 - j 6.7	34.1 + j 96.9	1.125	-25.0
0.2	7.04	6.02	18.5 - j 2.4	45.1 + j102.8	1.130	-28.3

Model RES-UR Element Length: Front: 0.2386 wl; Rear: 0.2505 wl
Frequency: 28.5 MHz Diameter: 0.001207 wl (0.5")

Space wl	Gain dBi	Front-to-Back Ratio dB	Z1 (Rear) R +/- jX Ohms	Z2 (Forward) R +/- jX Ohms	Rear I Magnitude	Rear I Phase
0.05	7.21	8.60	6.6 + j 16.5	-1.4 - j 8.8	0.950	- 7.8
0.075	7.33	7.80	8.8 + j 21.8	1.5 + j 2.8	0.950	-11.0
0.1	7.33	7.44	11.5 + j 27.0	5.9 + j 14.2	0.955	-14.5
0.125	7.30	7.13	14.1 + j 32.2	12.0 + j 24.4	0.960	-18.0
0.15	7.23	6.82	16.9 + j 37.3	19.6 + j 33.3	0.965	-21.5
0.175	7.15	6.36	19.0 + j 42.6	28.5 + j 39.7	0.960	-24.5
0.2	7.04	6.01	21.7 + j 47.6	37.9 + j 44.7	0.960	-27.8

Note: All gain values are for free-space. Rear current (I) magnitude and phase values are relative to forward element values of 1.0 and 0.0 degrees. Model RES-E uses elements of equal length to an independent resonant dipole at the test frequency. Models SHT-E and LNG-E uses elements that are 10% shorter and 10% longer, respectively. Models RES-UF and RES-UR uses elements that are 5% longer than those in RES-E at the forward and at the rear elements, respectively.

Table 3. Performance and operating conditions of 5 2-element phased arrays in a maximum-gain configuration.

Fig. 1-7 graphs the current magnitude and phase angle data for the 3 equal-element-length models. Once more the phase angle curves form an overlapping trio. Irregularities in the current magnitude curves arise from the simple averaging and centering procedure used to produce the curves. However, the general trend is both clear and consistent with the maximum front-to-back curves: the longer the elements, the higher the required relative current magnitude level on the rear element to achieve the desired performance curve.

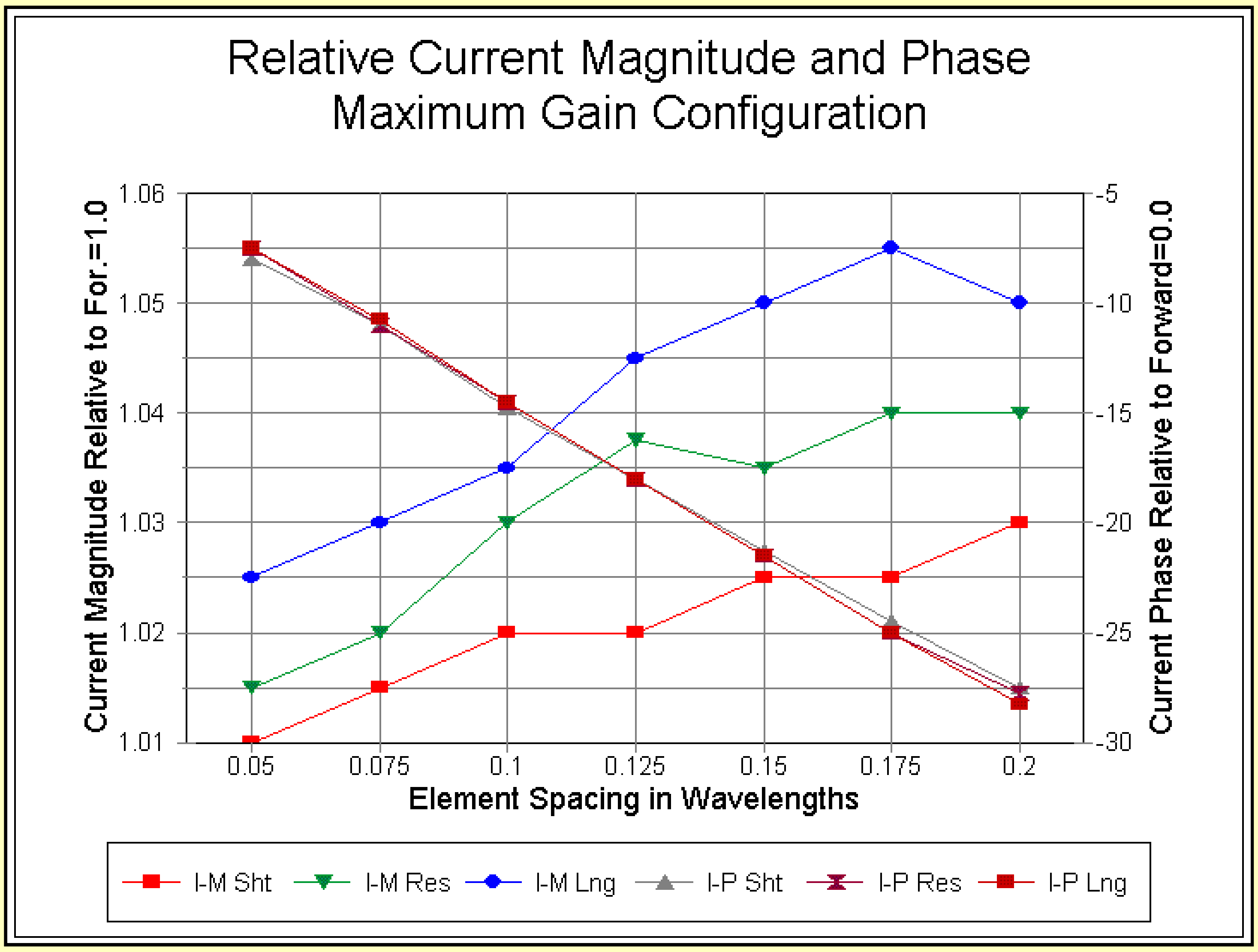

Fig. 1-7 The rear element relative current magnitude and phase angle for short, resonant, and long element lengths in arrays having equal- length forward and rear elements, and are set for maximum forward gain. "I-M" means rear element relative current magnitude. "I-P" means rear element relative current phase. "Sht" refers to model SHT-E; "Res" refers to model RES-E; and "Lng" refers to model LNG-E. See Table 3 for model specifications.

The maximum gain curves represent the highest gain level that we may achieve with 2 elements of the sizes in the models. In general, the highest gain levels coincide with those for a quite short boom 3-element Yagi or a 2-element quad, both of which are designed for adequate 10-meter band coverage. The Yagi boom length would be about 8' for this gain level, with 12' boom 3-element Yagis capable of 8 dBi free-space gain across the first MHz of 10 meters. However, the phased-array data, taken at a single frequency, do not necessarily hold over an equivalent operating bandwidth.

Conclusions and Compromise

The exercise that we have presented is at most a demonstration of phased array properties and not a proof of them. What it shows is two sets of limits between which most horizontal phased arrays operate. In general, designers either consciously select or discover through experimentation phasing arrangements that yield acceptable performance with respect to gain, front-to-back ratio, and operating bandwidth.

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        Performance Shifts With Changes in Relative Current Magnitude and Phase Angle
                              Model RES-E at 3 Element Spacings

Element Spacing: 0.05 wavelength
1.  Rear Element Relative Current Phase Angle:  -17.0 degrees
Rear I            Gain      Front-to-Back           Z1 (Rear)          Z2 (Forward)
Magnitude         dBi       Ratio dB                R +/- jX Ohms      R +/- jX Ohms
0.933             6.34      15.23                    3.7 - j 38.9       5.9 + j  6.1
0.983             6.47      21.48                    7.4 - j 37.0       2.2 + j  6.6
1.033             6.50      63.70                   10.7 - j 35.3      -1.6 + j  7.0
1.183             6.42      21.95                   13.7 - j 33.8      -5.3 + j  7.5
1.133             6.28      16.30                   16.4 - j 32.4      -9.0 + j  7.9

2.  Rear Element Relative Current Magnitude: 1.033
Rear I            Gain      Front-to-Back           Z1 (Rear)          Z2 (Forward)
Phase Angle       dBi       Ratio dB                R +/- jX Ohms      R +/- jX Ohms
-13.0             6.88      17.45                    8.5 - j 30.7      -2.0 + j  1.6
-15.0             6.69      23.97                    9.6 - j 33.1      -1.8 + j  4.3
-17.0             6.50      63.70                   10.7 - j 35.3      -1.6 + j  7.0
-19.0             6.30      25.07                   11.9 - j 37.6      -1.2 + j  9.7
-21.0             6.11      19.47                   13.2 - j 39.8      -0.7 + j 12.4

Element Spacing: 0.125 wavelength
1.  Rear Element Relative Current Phase Angle:  -44.7 degrees
Rear I            Gain      Front-to-Back           Z1 (Rear)          Z2 (Forward)
Magnitude         dBi       Ratio dB                R +/- jX Ohms      R +/- jX Ohms
0.974             6.31      23.22                   20.3 - j 42.5      33.8 + j 46.2
1.024             6.33      29.53                   22.8 - j 40.5      31.8 + j 48.7
1.074             6.33      64.42                   25.1 - j 38.8      29.8 + j 51.2
1.124             6.31      29.77                   27.2 - j 37.1      27.9 + j 53.7
1.174             6.28      24.03                   29.1 - j 35.7      25.9 + j 56.2

2.  Rear Element Relative Current Magnitude: 1.074
Rear I            Gain      Front-to-Back           Z1 (Rear)          Z2 (Forward)
Phase Angle       dBi       Ratio dB                R +/- jX Ohms      R +/- jX Ohms
-40.7             6.52      25.75                   22.7 - j 35.4      26.2 + j 48.1
-42.7             6.42      31.85                   23.9 - j 37.1      28.0 + j 49.7
-44.7             6.33      64.42                   25.1 - j 38.8      29.8 + j 51.2
-46.7             6.23      32.45                   26.4 - j 40.4      31.7 + j 52.7
-48.7             6.14      26.50                   27.8 - j 41.9      33.7 + j 54.0

Element Spacing: 0.2 wavelength
1.  Rear Element Relative Current Phase Angle:  -73.6 degrees
```

Rear I Magnitude	Gain dBi	Front-to-Back Ratio dB	Z1 (Rear) R +/- jX Ohms	Z2 (Forward) R +/- jX Ohms
0.980	5.76	26.01	32.5 - j 41.3	80.3 + j 51.3
1.030	5.76	32.36	34.3 - j 39.4	80.8 + j 54.0
1.080	5.76	63.34	36.0 - j 37.6	81.3 + j 56.7
1.130	5.76	32.38	37.5 - j 36.0	81.8 + j 59.3
1.180	5.75	26.68	38.9 - j 34.5	82.3 + j 62.0

2. Rear Element Relative Current Magnitude: 1.080

Rear I Phase Angle	Gain dBi	Front-to-Back Ratio dB	Z1 (Rear) R +/- jX Ohms	Z2 (Forward) R +/- jX Ohms
-69.6	5.92	28.63	33.6 - j 35.1	77.3 + j 57.3
-71.6	5.84	34.61	34.8 - j 36.4	79.3 + j 57.0
-73.6	5.76	63.34	36.0 - j 37.6	81.3 + j 56.7
-75.6	5.69	35.05	37.3 - j 38.8	83.3 + j 56.3
-77.6	5.61	28.98	38.7 - j 39.9	85.3 + j 55.8

Note: Total rear element relative current magnitude shift: +/- 10%; total rear element relative current phase angle shift: +/- 2 degrees

Table 4. Performance shifts in model RES-E at 0.05, 0.125, and 0.2 wavelength element spacing with a constant rear element relative phase angle and a variable relative current magnitude and with a constant rear element current magnitude and a variable relative current phase angle.

Table 4 gives us a partial view of what happens to the performance characteristics of a 2-element array as we drift away from the conditions that yield maximum front-to-back ratio. Varying the rear element relative current magnitude alone (with a fixed relative current phase angle) by about +/- 10% shows a gradual decline in gain and a more rapid decrease in front-to-back ratio whether the current magnitude goes too high or too low. However, as we fix the current magnitude on the rear element and vary the phase angle, we obtain a different progression. The front-to-back ratio decreases on both sides of the optimal values. However, the change in phase angle shows a single low-to-high progression in the +/-2 degree variation in the example.

The table shows clearly that the operating bandwidth for a set of conditions varies directly with the spacing between elements. The cost of obtaining the wider operating bandwidth is, of course, a decrease in the forward gain. However, the rate of gain decrease itself increases with spacings above about 0.125 wavelength. Indeed, one of the sensible reasons for selecting an element spacing in the 0.1 to 0.15 wavelength region is that we acquire reasonable operating bandwidth while maintaining higher gain levels.

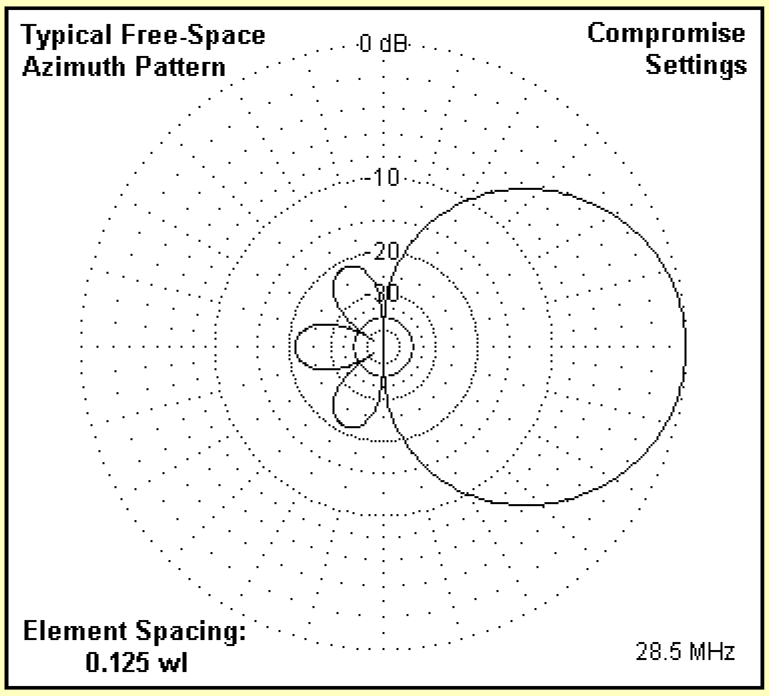

Fig. 1-8 Typical "desirable" free-space azimuth pattern for a 2-element horizontal phased array set for acceptable amateur operation.

Designers of phased arrays rarely survey the potentials for practical beams by extending the systematic model variation exemplified by **Table 4**. There are too many variables involved in the design work for one to fix upon a set of relative current magnitudes and phase angles and then design means for obtaining them. Instead, they tend to discover configurations that meet our usual amateur standards for what counts as a "good" beam. **Fig. 1-8** shows a typical and desirable phased array pattern for an array using equal length (self-resonant) elements and spaced 0.125 wavelength. Gain does not appear on the pattern, but the triple rear lobe everywhere exceeds -20 dB relative to the forward lobe.

Performance Shifts With Changes in Relative Current Magnitude and Phase Angle
Model RES-E at 0.125 wavelength Element Spacing Stepped Between Front-to-Back and Gain Settings

Setting No.	Rear I Mag.	Rear I Phase	Gain dBi	Front-to-Back Ratio dB	Z1 (Rear) R +/- jX Ohms	Z2 (Forward) R +/- jX Ohms
1	**1.074**	**-44.7**	**6.33**	**64.42**	**25.1 - j 38.8**	**29.8 + j 51.2**
2	**1.065**	**-38.0**	**6.64**	**21.05**	**20.8 - j 33.3**	**24.2 + j 45.5**
3	**1.056**	**-31.4**	**6.94**	**14.53**	**17.1 - j 27.4**	**19.5 + j 39.3**
4	**1.047**	**-24.7**	**7.17**	**10.32**	**14.1 - j 21.0**	**15.4 + j 32.6**
5	**1.038**	**-18.0**	**7.28**	**7.08**	**11.8 - j 14.2**	**12.3 + j 25.5**

Table 5. Performance shifts as the relative rear element current magnitude and phase angles are shifted in proportional steps between maximum front-to-back ratio and maximum gain settings.

There is no single set of values for relative current magnitude and relative phase angle that will yield patterns of this sort. **Table 5** lists data for a set of compromise values developed simply by taking proportional parts of the differentials between the magnitude and phase angle values for the two extreme or limiting cases. **Fig. 1-9** graphs the free-space gain and front-to-back ratio. The setting numbers correspond to the combinations shown in the table.

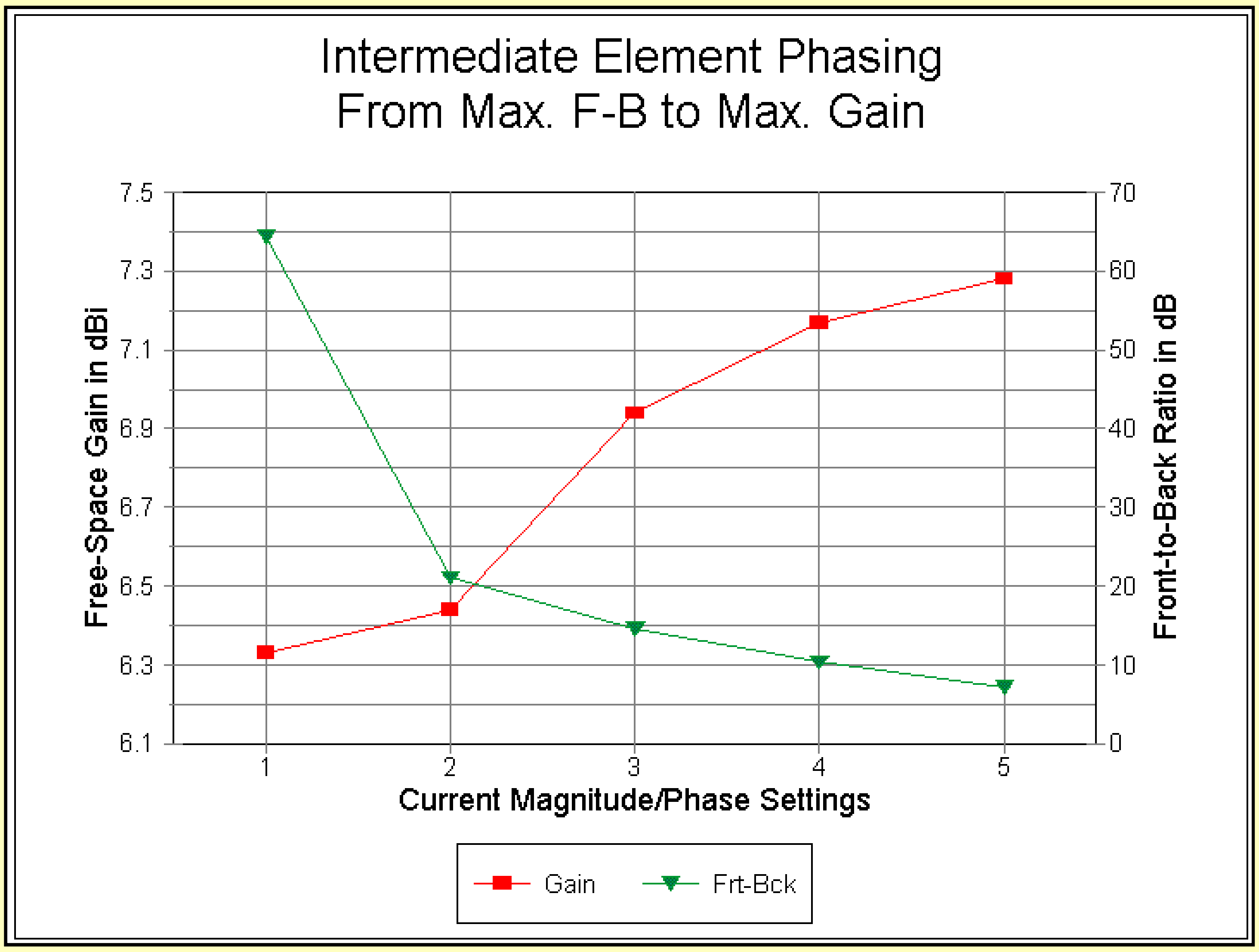

Fig. 1-9 Free-space gain and front-to-back ratio of 2-element horizontal phased arrays at compromise settings of rear element relative current magnitude and phase angles between the limits of maximum forward gain settings and maximum front-to-back ratio settings. See Table 5 for details of the compromise settings.

As noted earlier, the very high 180-degree front-to-back ratio decreases quickly, so that a phase angle of -38 degrees on the rear element with a 1% decrease in current magnitude results in a front-to-back ratio just over 20 dB. However, in this increment, gain only rises by about 0.1 dB, with the steeper gain increase curve appearing between settings 2 and 3. As a result, one must accept a front-to-back ratio of less than 20 dB to achieve gain levels higher than 6.5 dBi.

The strategy used for these models can well be altered with possibly different results. We have sampled only two of many strategies in the effort to find a satisfactory set of operating conditions, and we have not explored the question of operating bandwidth--the frequency range over which the performance characteristics sustain themselves at acceptable levels. One reason for this void in our discussion is that the means by which we effect the current magnitudes and phase angles on each element play a significant role in setting the operating bandwidth. The exploration of such means is yet to come. We can only note at this stage that the number of variables involved in phased array design is high enough to preclude anything like a complete treatment.

So far, we have only scratched the surface of horizontal array understanding. The exercise has set performance limits. The data in **Tables 1 and 2**, however, are more than interesting numbers: they provide insight into the conditions that yield individual element impedances in paired combinations. The pattern of impedances will take on considerable importance in Parts 3 and 4 of this series.

As well, we have identified some of the factors affecting operating bandwidth, such as element spacing and where we set the rear element relative current magnitude and phase angle between the maximum gain and the maximum front-to-back values. Of course, we have not mentioned a third significant factor that affects operating bandwidth, namely, the diameter of the elements that we use. However, element diameter as a fraction of a wavelength will play a role in operating bandwidth, especially as one examines wire and tubular implementations of 2-element phased arrays.

So far, we have not explored how close we may come to a nearly perfect array with the ordinary design means available to us. One of those ordinary means that we usually overlook is antenna geometry. We shall explore the nature and limitations of that design route in the next episode.

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