Stacking: What Difference Does Difference Make?

L. B. Cebik, W4RNL (SK)

Dave Leeson recently sent me a quandary concerning the effect that a beta match inductive reactance might have on a stack of 2 (or more) antennas. The beta match, of course, is an L-network, and network outputs have their voltage and current shifted in phase relative their input phases. If two identical beta-matched antennas are stacked and are fed exactly in phase, then the network phase shifts would be equal, and the two antenna element arrays would also be in phase with each other.

However, discussions in various quarters have suggested that many hams are stacking different antennas, each ostensibly for the same band. I have a feeling in retrospect that this may have been what Dave had in mind when he raised the question. The question in fact is not a simple one and may have at least two dimensions. On the one hand, some hams are stacking antennas having different forward gain values. On the other, they may be stacking antennas having different matching systems to yield a terminal 50-Ohm impedance.

Together, the two questions present an almost endless array of combinations for any possible installation. Therefore, no definitive set of answers can be generated for a particular installation without extensive measurement at the site itself. Nevertheless, modeling can indicate some suggestive directions for the individual stacker to investigate if he is concerned about such a situation.

Let's divide the questions into two parts and look first at the differential gain situation.

Differential Gain Antennas in a Stack

If we place two antennas of different gain values in a stack, how will they perform? What will be the maximum gain achievable from the stack? What will be the take-off angle (elevation angle of maximum radiation)?

To get an initial purchase on this question, I took two 20-meter 3-element beams out of my files. One is a short boom (16') model with a little over 7 dBi free space gain at 14.175 MHz. The other is a longer-boom (24') model with a little over 8 dBi free space gain at the same frequency. I placed each over average ground at a height of 70' (a rounded 1 wl). The basic single antenna performance at that height for each was as follows:

Antenna	**Gain 70' dBi**	**TO 70' deg**	**F-B 70' dB**
#1: 3-el/16' boom	**12.56**	**14**	**34.97**
#2: 3-el/24' boom	**13.39**	**14**	**25.05**

I next created stacks of two identical antennas of each kind. I sought out the separation above the 70' height of the lower antenna that would yield maximum gain, without regard for front-to-back ratio. In some cases, only a single height for the second antenna produces maximum gain--in other cases, the maximum gain may be spread over several feet of differential separation. For the two antennas, here are the results:

2 x Antenna #1: lower antenna at 70'

Height upper	**space in ft**	**space in wl**	**Gain**	**TO**	**F-B**
115'	**45'**	**.65**	**15.59**	**10**	**15.69**

2 x Antenna #2: lower antenna at 70'

Height upper	**space in ft**	**space in wl**	**Gain**	**TO**	**F-B**
115'	**45'**	**.65**	**16.21**	**10**	**17.52**
116'	**46'**	**.66**	**16.21**	**10**	**17.56**
117'	**47'**	**.68**	**16.21**	**10**	**17.69**

For the lower gain antenna, the net stacking gain is 2.03 dB, while for the higher gain antenna, the net stacking gain is 1.82 dB. The upper antenna alone exhibits a marginally lower TO angle, but a more significant reduction in gain. For example, Antenna #2, the higher gain model, when placed at 116', shows a gain of 13.71 dBi with a TO angle of 8 degrees. Of course, it maintains a high front-to-back ratio, in this case 27.4 dB.

Now suppose we combine one antenna of each type. We have a choice to make as to which of the two goes on top. This choice may be governed by other than performance figures. Weight distribution on a mast and similar installation considerations may in fact outweigh the performance numbers. However, it is interesting to discover what those numbers might be, at least in modeled form.

The two situations were similarly modeled, placing first one and then the other on top. Throughout, I assumed that the two could be truly fed in phase in order to keep that other question separated from this one. As with the stack of two identical beams, I sought out the heights/separations that yielded maximum gain without reference to the consequences on the front-to-back ratio. Here are the results:

Antenna #1 low--Antenna #2 high: lower antenna at 70'

Height upper	**space in ft**	**space in wl**	**Gain**	**TO**	**F-B**
118'	**48'**	**.69**	**15.79**	**9**	**17.70**
119'	**49'**	**.71**	**15.79**	**9**	**17.95**
120'	**50'**	**.72**	**15.79**	**9**	**18.25**
121'	**51'**	**.74**	**15.79**	**9**	**18.60**

Antenna #2 low--Antenna #1 high: lower antenna at 70'

Height upper	**space in ft**	**space in wl**	**Gain**	**TO**	**F-B**
116'	**46'**	**.66**	**15.61**	**10**	**16.34**

117'	**47'**	**.68**	**15.61**	**10**	**16.39**
118'	**48'**	**.69**	**15.61**	**10**	**16.50**

Placing the higher gain antenna on top of the stack above the lower gain antenna requires greater separation than simply stacking two of the higher gain antennas. The result is a TO angle that is a degree lower than stacks of two identical antennas. However, the gain is intermediate between the two stacks of identical antennas.

Placing the lower gain antenna on top requires at least as much separation as a stack of two identical higher gain antennas. Yet, the peak gain is not significantly different than a stack of two of the lower gain antennas.

The upshot is a suggestive conclusion--not by any means a firm, let along a decisive one: Stacking beams of dissimilar gain is a useful practice, although the ultimate gain of the system will be closer to the value for two of the lower gain antennas rather than to the value for two of the higher gain antennas. Wherever possible, the higher gain antenna deserves to be on top to maximize performance from the pair, whether or not in practical terms it can in fact be on top. Expect maximum gain to occur with slightly greater spacing than might be used for identical beams.

Confirmation of these conclusions would require very extensive modeling just to reach the stage of a firm conclusion. In the absence of such a large scale systematic study, every potential stacker should equip himself to analyze his own situation in detail, including that very elusive task of arriving at a reliable figure for each of the commercial antennas involved in the proposed stack.

The Feed-Phase Question

The initial foray into stacking dissimilar gain antennas assumed in phase feeding without further question. What happens when the antennas see differential phasing on their drive elements? This question is an extremely complex one, because both the magnitude and the phase of the source current may differ between two antennas. Since the combinations one might try are nearly endless, we shall have to settle once more for a suggestive answer dealing only in the simplest case.

Let us assume that the current magnitude at the antenna sources are equal in all cases. The lower antenna of a stack of two identical antennas (the higher gain model #2 above) will have an arbitrary current magnitude of 1.0 at a phase angle of 0.0 degrees. The spacing of the antennas is set by the upper antenna also having the same source current magnitude and phase. In this case, the spacing is 46'. Now we can address the question of what happens if the phase angle of the source current varies from that of the fixed lower antenna values.

It is not possible in advance to say what a likely region of a possible 359 degree variation in current phase angles might be. For example, two beta matches might be set slightly different to each other, perhaps the result of using slightly different lengths for the driver. The consequence might be a few degrees difference in feed current phase angle. At the other extreme, one might combine two different antennas (of equal gain) having radically different matching systems and thus creating a large difference in the source current phase at the antenna elements.

The only practical solution for a first order look at the possible problem is simply to vary the current phase at the source of the upper antenna in regular increments and see what happens. Therefore, I modeled the system of identical antennas simply by changing the source current phase in 10- degree increments. The following long table is what I obtained by way of modeling output.

Table of Feed Phase Differences
Imag = 1.0/1.0 Iphase = 0.0/X degrees

A. Reference:

1 Antenna	**Gain dBi**	**TO deg**	**2nd lobe dBi**	**TO deg**	**F-B dB**	**Zlower R/jX**	**Zupper**
70'	**13.39**	**14**	**---**	**--**	**25.1**	**25.1-0.9**	**---**
116'	**13.71**	**8**	**---**	**--**	**27.4**	**25.9-0.6**	**---**

B. Systematic Variation of the Top Antenna Source (2) Current Phase

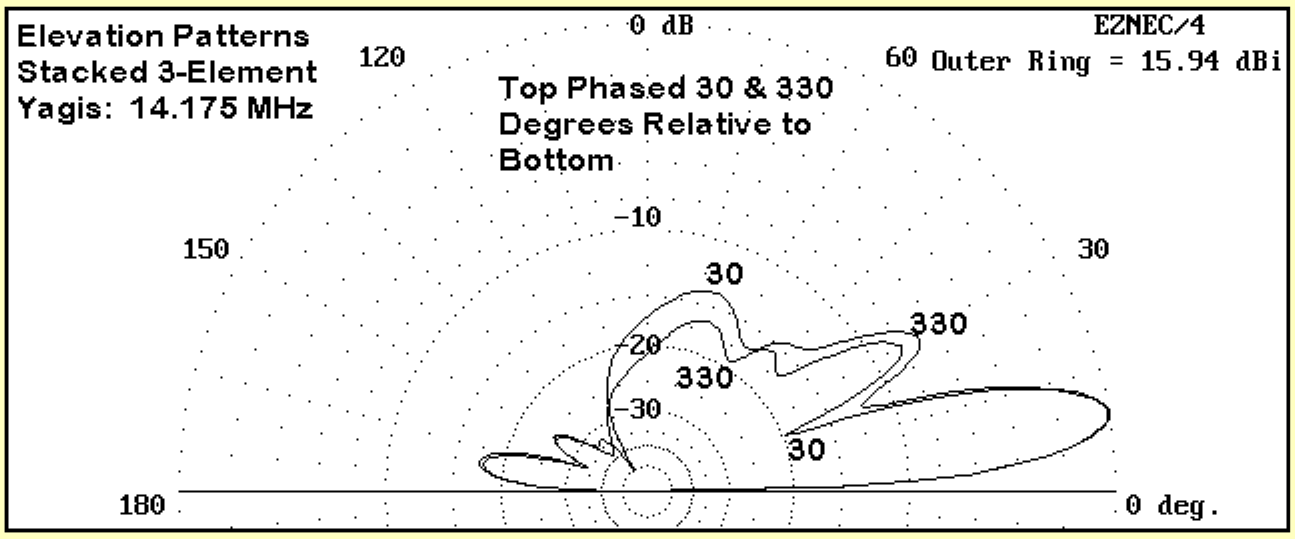
Source 2 Phase	**Gain dBi**	**TO deg**	**2nd lobe dBi**	**TO deg**	**F-B dB**	**Zlower R/jX**	**Zupper R/jX**
0	**16.21**	**10**	**---**	**---**	**17.6**	**24.1+0.9**	**24.8+1.2**
10	**16.17**	**10**	**---**	**---**	**17.5**	**23.7+0.8**	**25.2+1.2**
20	**16.06**	**10**	**---**	**---**	**16.1**	**23.4+0.7**	**25.6+1.2**
30	**15.88**	**10**	**---**	**---**	**17.3**	**23.1+0.5**	**25.9+1.0**
40	**15.63**	**10**	**---**	**---**	**17.3**	**22.8+0.3**	**26.2+0.8**
50	**15.30**	**10**	**---**	**---**	**17.2**	**22.6-0.0**	**26.5+0.6**
60	**14.89**	**9**	**---**	**---**	**17.2**	**22.4-0.3**	**26.7+0.3**
70	**14.41**	**9**	**---**	**---**	**17.1**	**22.2-0.7**	**26.9+0.0**
80	**13.82**	**9**	**---**	**---**	**17.0**	**22.2-1.0**	**27.0-0.3**
90	**13.13**	**9**	**---**	**---**	**16.9**	**22.2-1.4**	**27.1-0.7**
100	**12.30**	**9**	**---**	**---**	**16.8**	**22.2-1.8**	**27.1-1.1**
110	**11.32**	**9**	**---**	**---**	**16.7**	**22.3-2.1**	**27.0-1.4**
120**	**11.41**	**47**	**11.19**	**24**	**---**	**22.5-2.4**	**26.9-1.8**
130	**11.81**	**46**	**11.58**	**24**	**---**	**22.8-2.7**	**26.7-2.1**
140	**12.13**	**46**	**11.92**	**23**	**---**	**23.0-2.9**	**26.5-2.4**
150	**12.39**	**46**	**12.21**	**23**	**---**	**23.4-3.1**	**26.2-2.6**
160	**12.57**	**46**	**12.43**	**23**	**---**	**23.7-3.3**	**25.9-2.8**
170	**12.68**	**46**	**12.58**	**23**	**---**	**24.1-3.3**	**25.5-2.9**
180	**12.73**	**46**	**12.66**	**23**	**---**	**24.4-3.3**	**25.2-3.0**
190	**12.71**	**46**	**12.69**	**23**	**---**	**24.8-3.3**	**24.8-3.0**
200**	**12.65**	**23**	**12.63**	**46**	**---**	**25.1-3.2**	**24.4-2.9**
210	**12.54**	**23**	**12.48**	**46**	**---**	**25.5-3.0**	**24.1-2.8**
220	**12.38**	**23**	**12.26**	**46**	**---**	**25.7-2.7**	**23.8-2.6**
230	**12.14**	**23**	**11.96**	**46**	**---**	**26.0-2.5**	**23.5-2.3**
240	**11.83**	**23**	**11.59**	**46**	**---**	**26.2-2.1**	**23.3-2.1**
250**	**11.53**	**10**	**---**	**---**	**18.9**	**26.3-1.8**	**23.1-1.7**
260	**12.49**	**10**	**---**	**---**	**18.7**	**26.4-1.4**	**23.0-1.4**
270	**13.30**	**10**	**---**	**---**	**18.5**	**26.4-1.1**	**22.9-1.0**
280	**13.98**	**10**	**---**	**---**	**18.3**	**26.3-0.7**	**22.9-0.7**
290	**14.58**	**10**	**---**	**---**	**18.2**	**26.2-0.4**	**23.0-0.3**
300	**15.02**	**10**	**---**	**---**	**18.1**	**26.0-0.0**	**23.1+0.0**
310*	**15.41**	**10**	**---**	**---**	**18.0**	**25.8+0.2**	**23.3+0.4**

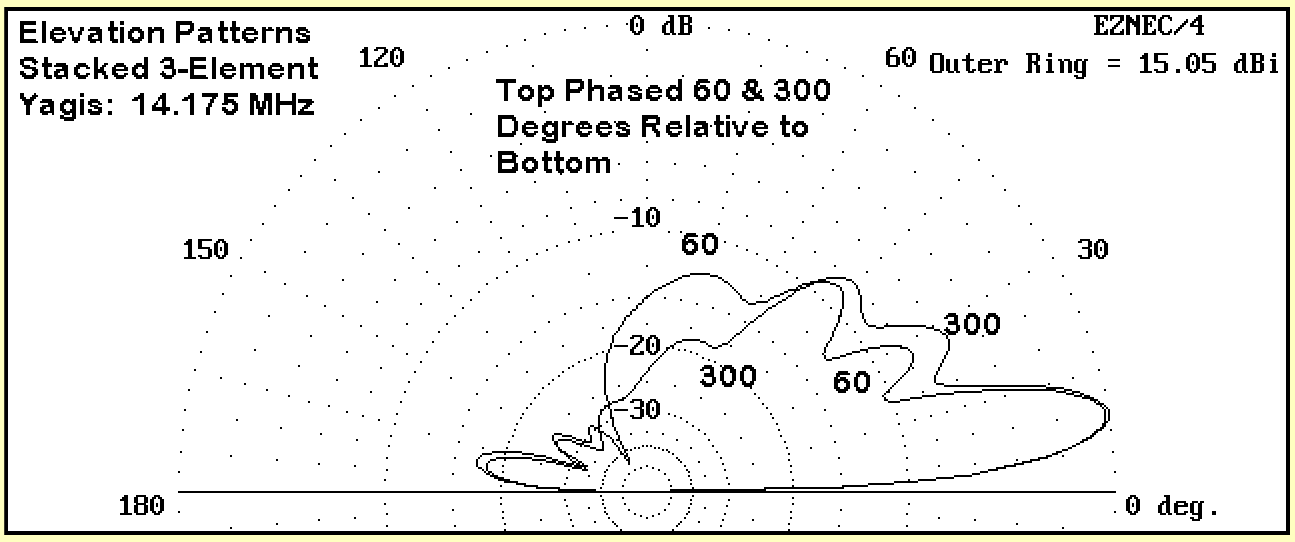
320*	**15.71**	**10**	**---**	**---**	**17.9**	**25.5+0.5**	**23.5+0.6**
330*	**15.94**	**10**	**---**	**---**	**17.8**	**25.2+0.7**	**23.8+0.9**
340*	**16.10**	**10**	**---**	**---**	**17.7**	**24.8+0.8**	**24.1+1.0**
350	**16.19**	**10**	**---**	**---**	**17.7**	**24.5+0.9**	**24.5+1.2**
360	**16.21**	**10**	**---**	**---**	**17.6**	**24.1+0.9**	**24.8+1.2**

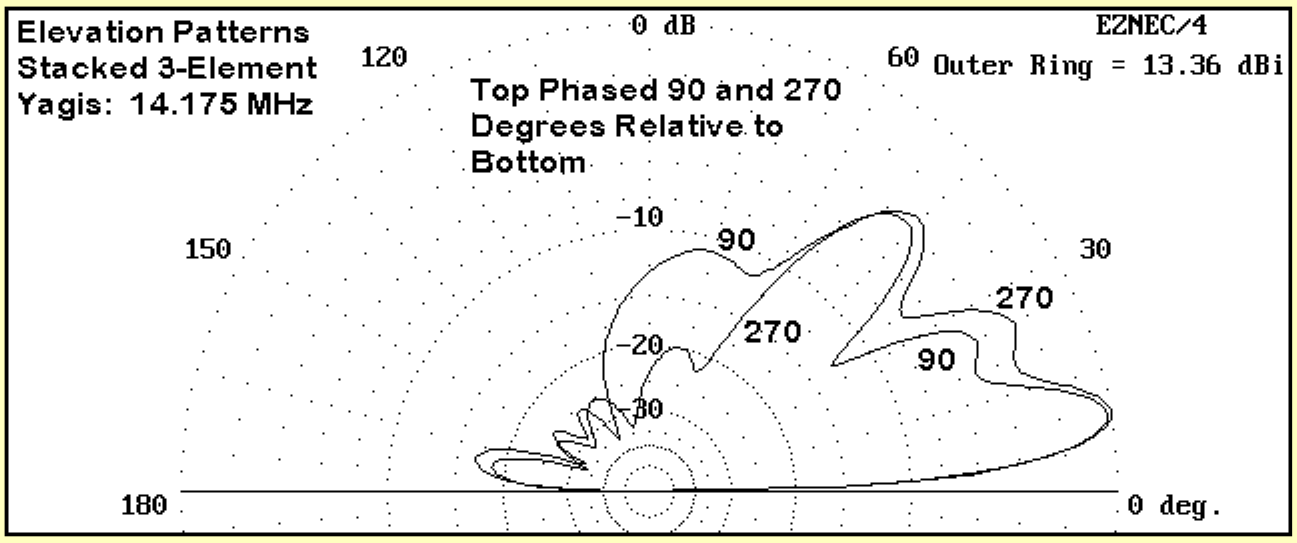
The ** indicator shows where the lobe of maximum gain undergoes a large shift. When the angle of maximum radiation is high, I have ignored front- to-back figures and given instead the strength of the accompanying secondary lobe, which is ordinarily notably strong relative to the primary lobe. Where the main lobe is at a low angle, front-to-back is given. In cases of a low-angle main lobe, I ignored the secondary lobe except to note some instances where it was very weak, as indicated by the single *.

Most notable in the current phase circle is that for over a third of the possible values, the main and secondary lobes are angled too high for effective DX use. For over three-fourths of the values, the forward gain is less than that of a single antenna at the lower height of the stacked pair. The window of maximum gain, arbitrarily defined here as within 0.25 dB of the highest value of which the system is capable is +/-20 degrees of a perfect in-phase condition. Doubling the margin of acceptability to 0.5 dB down from peak capability increases the range of acceptable phase differentials to +/- 40 degrees.

Although there is a certain comparability as we move equal amounts plus and minus of a perfect in-phase condition, there is no symmetry. The following three elevation overlays--taken at +/-30, +/-60, and +/-90 degrees provides ample evidence of the differences.







This single demonstration is but one of innumerable others that might run the same ring with the top antenna having more or less than equal current magnitude relative to the bottom antenna (which is fixed at a current magnitude of 1.0 and a phase of 0.0 degrees for the exercise). To reveal the most optimal set(s) of conditions between the two antennas in the stack would require a catalog of runs. For values of top-beam source current magnitude from 0.9 through 1.10 relative to that of the bottom beam, a perfect in-phase condition between the two currents does yield maximum system gain under the prescribed set-up of the stack used in this sample exercise.

Current Magnitude and Phase

Dean Straw, N6BV, recently (May, '99) reminded me that the current magnitude cannot be ignored in these calculations, and, of course, he is correct. A source connected on the input side of a beta (or other) match does not yield a current equal to the source current on the antenna side of the match. Hence, not only will the phase of the current shift, but so too will its magnitude. If we place antennas with different matching parameters into a stack, the antennas may produce something other than peak performance. At the time I worked up the original note, I did not have models of sufficiently similar operating characteristics to demonstrate this point. I have since acquired some.

To make a practical test of this within the models at hand, I took two different antennas of approximately the same capability. The models are fully described in another note, Modeling 6 Long-Boom Yagis. The Yagis selected include the one designated as K6STI, a design developed by N6BV on YO for the YA program. It has a free space gain at 14.175 MHz of 10.52 dBi (NEC-4 uncorrected) with a front-to-back ratio of 23.64 dB. The native source impedance is 23.6 - j27.6 Ohms.

The other Yagi selected is the W3LPL design, but without the Tee match used by the designer. Its free space gain at 14.175 MHz is 10.30 dBi, with a front-to-back ratio of 22.80 dB (NEC-4 uncorrected). The feedpoint impedance of the design as given is 39.1 + j17.9 Ohms. In performance, at the selected frequency, the beams would be indistinguishable except for the feedpoint impedance.

My first step was to reduce the length of the W3LPL driver until the source impedance showed a considerable capacitive reactance. The resultant source impedance was 31.5 - j23.6 Ohms with no change of performance. I then applied beta match shorted stubs to the source segments of each antenna: 48 Ohms (101" shorted stub of 50-Ohm, VF=1 perfect transmission line) for the N6BV/K6STI beam and 65 Ohms (122" shorted stub of 50-Ohm, VF=1 perfect transmission line) for the W3LPL Yagi. These matches brought each antenna to under 1.2:1 50-Ohm VSWR.

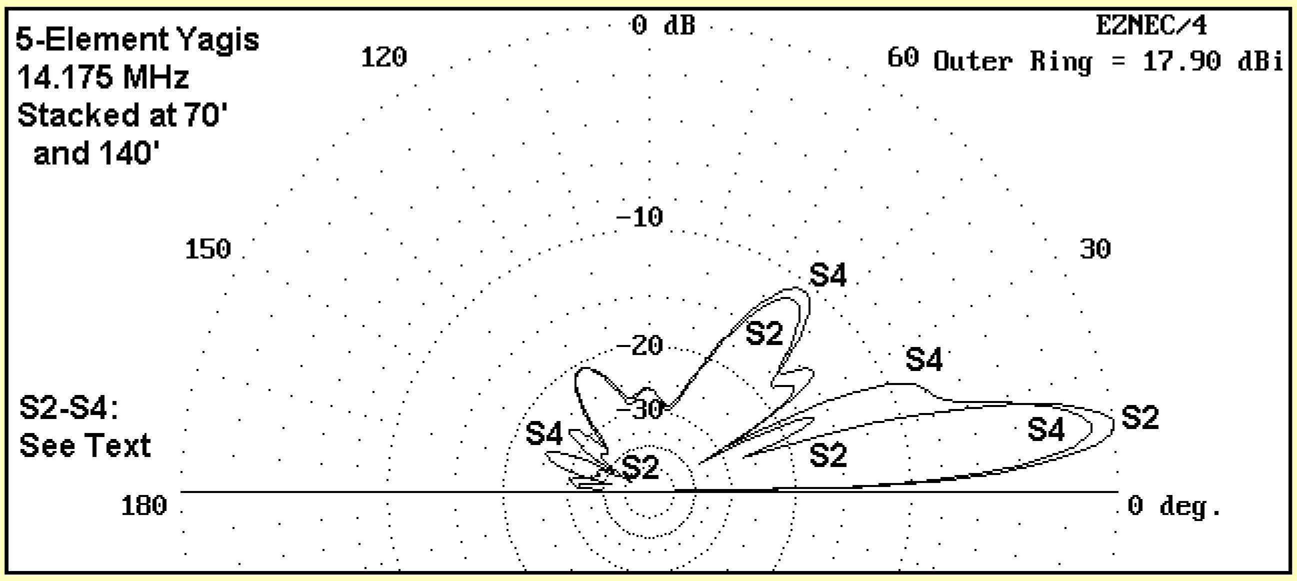
Next, I created a stack with the N6BV/K6STI beam at 70' and the W3LPL antenna at 140' above average ground. Each antenna source was connected to a common (remote modeling) wire through 1/4 wl sections of 75-Ohm transmission line to effect an impedance transformation to about 100 Ohms. The parallel combination should yield a net impedance very close to 50 Ohms for connection to a main feedline.

The stack produced a reported gain of 17.90 dBi at TO angle of 8 degrees, with a front-to-back ratio of 30.32 dB. The feedpoint impedance of the combination was 52.6 + j 1.1 Ohms. The currents for the antenna driver feed segments are (relative to a source current of magnitude 1 and phase angle 0) as follows: Lower: 1.11 at -39.3 degrees; Upper: 0.89 at -52.3 degrees. The net difference in magnitude is 0.22 and the difference in phase is 13.0 degrees. These are fairly small differences which do not detract significantly from peak performance of the array. (Two in-phase-fed W3LPL beams stacked at 70' and 140' show a maximum gain of 17.91 dBi, while two of the N6BV/K6STI beams in the same position show a maximum gain of 18.05 dBi.)

Next I created for the W3LPL beam in its original configuration (an inductively reactive driver) a -103-Ohm (capacitive) reactance open stub from 60" of open 50-Ohm perfect line. (Although rarely used, a beta match may employ a series inductive reactance and a shunt capacitive reactance.) With this open stub shunt, the performance characteristics remained as before, but the feedpoint impedance was altered to 47.3 - j0.2 Ohms. I then replaced the upper beam in the 70-140 stack with the new version of the W3LPL antenna and reconnected the feed system used in the first test.

The new stack, which is identical to the old in every way except the driver matching scheme of the upper antenna, produced a reported gain over average ground of 17.05 dBi, again at an 8-degree TO angle, with a front-to-back ratio of 33.20 dB. The combined net source impedance was 54.9 - j0.8 Ohms for a 50-Ohm VSWR figure of under 1.1:1.

The relative source segment currents were as follows: Lower: 1.13 at -39.5 degrees; Upper: 0.79 at -113.6 degrees. The differential amount to 0.34 in magnitude and 74.1 degrees in phase. The consequence is a loss of nearly a dB in gain from the array relative to the more closely matched first test configuration. Moreover, there are noticeable differences in the elevation patterns of the two configurations, as shown in Fig. 4.



S2 represents the stack with the two beta matches that use shorted stubs. S4 represents the combination of shorted lower and open upper stub beta matches. Note not only the gain of the lowest lobe, but as well the rear quadrant. Although the lower gain combination has the smaller rear lobe at the first level, its upper rear lobes are significantly larger than those of S2. Note also the second forward lobe of S4, which blends into the lower lobe.

As Dean noted to me, there are still antennas around with native source impedances between 12 and 20 Ohms, and these will show even wider variations in current magnitude. Unfortunately, I do not have in my collection any models that match up well in performance characteristics to those with higher source impedances. Hence, a modeling exercise would not be able to separate the consequences of current magnitude and phase differences from the consequences of performance differences. Nevertheless, the small demonstration given here should suffice to indicate in a broad way that current magnitude and phase together can have an impact on stack performance, even if the antennas stacked are very closely matched in performance characteristics. The wider the difference of source segment current magnitude and phase, the more likely the stack is to perform at less than its full potential.

A Short Note on Networks

The phase shift created by networks between the feeding phase lines and the post-network antenna terminals depends on the nature of the network and the conditions under which it is operating. The latter conditions include both the complex impedance transformation involved and the "delta" of the network. L-networks--with two reactance arms--are limited to phase shifts of +/- 90 degrees, while the 3-armed PI and Tee networks may have any desired phase shift value. Very often, one designs for the impedance transformation and accepts whatever phase shift results. This result is not a problem for a single network feeding a single antenna as its load. Since this is the standard case, introductions in ham literature often stress the conditions of impedance transformation and efficiency, while bypassing significant notice of the conditions of phase shift. Phase shift information tends to appear in antenna manuals, especially in chapters devoted to the "phasing" of vertical antenna elements in an array.

The current literature and techniques of network analysis often begin at a level somewhat daunting to the average ham who is simply trying to design an antenna installation using commercially-made components. However, the fundamental relationships for the most common network can be found in Terman, ***Radio Engineers' Handbook***, Section 3, "Paragraph" 25 (pp. 210-215 of the 1943 edition). Equations 113 and 114 are useful for getting a "feel" for how the basic elements of the network interrelate in Tee and PI networks. The preceding paragraph is drawn from this section. Those already beyond this introductory level can amend and supplement the treatment in many ways, especially in showing the PI, Tee, and L networks to be special cases of more general principles.

Phase shifts in networks can be calculated via a number of available utility programs. TLA, by N6BV, will handle networks as well as transmission lines when used rightly. There are also utilities in the HAMCALC collection to perform similar functions. And these are just two of a number of sources.

Impedance transformation, efficiency, and phase shift are interrelated determinable conditions for all common types of networks, but not necessarily easily determined conditions for any particular installation. Perhaps in the end, the most common advice is the soundest: in the absence of detailed measurements and calculations of the post-network antenna terminal current phase conditions, strive for identity between the antennas in the stack.

These notes are intended only to suggest the dimensions of an aspect of antenna stacking. In no way do they pretend either to solve a problem or to create one where one does not presently exist for a given installation. Hopefully, they may contribute at an introductory level in a better understanding of what stacking may involve when the antennas stacked are not identical either in gain or in the current magnitude and phase shift created by an impedance transformation network incorporated into the antenna feedpoint system.

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