Supplementary Notes on Stacking

L. B. Cebik, W4RNL (SK)

Background

In March of 1997, Roger Cox, WB0DGF, of Telex/Hy-Gain sent me a table of recommended stacking heights (spacing between beams in a 2-beam vertical stack) based on extensive modeling he had done. Roger's work showed that for maximum gain from the stack (which often does not coincide with maximum front-to-back ratio), the separation required depends almost wholly on the initial gain of the single antennas in the stack. His table provided a range of maximum array gain separation heights for his subdivisions in the original antenna gains he studied.

This discovery, apparently first made a number of years ago, has an origin unknown to me. Someone suggested that it appeared in older editions of the ARRL antenna book, but a search of the seven editions on my shelf turned up nothing. The data remain relatively unknown and unappreciated by hams, but deserve preservation, and hence, I am adding this note to my collection to make the information more readily available.

Almost any antenna modeler can confirm for himself or herself the general trends noted by Roger. In fact, Tom Schiller, N6BT, of Force 12 sent me a very similar table. Naturally, those involved in amateur antenna design would use antenna modeling software to confirm the original findings, using their own antenna designs as test subjects. The table Tom sent me reads as follows:

Free space Gain in dBi	# of Elements	Boom Length wl / ft	Vertical Spacing for Max Gain (wl)
6.6	2	0.15 / 10.5	0.4 - 0.6
7.6	3	0.25 / 17.5	0.5 - 0.7
8.8	4	0.43 / 30.1	0.7 - 0.9
9.9	5	0.60 / 42.0	0.9 - 1.1
10.5	6	0.75 / 52.5	1.0 - 1.1
11.6	8	1.10 / 77.0	1.3 - 1.4

Why this phenomenon occurs is not hard to understand. The easiest way to get a handle on what is happening is to look at overlays of far field patterns in free space for both low gain and high gain antennas. You will notice that the H-plane pattern (the elevation pattern in conventional modeling graphics) of the high gain antenna is pushed considerably forward relative to the low gain antenna. Now this use of far-field patterns is not strictly correct (which is why I do not introduce a graphic here), since the immediate field surrounding the antenna elements in the stack is more complex. However, it would be a routine, although drudgery-filled task to calculate the height differences necessary for these displaced fields to join in phase for maximum additive field strength. The more forward the individual patterns, the greater the height necessary for maximum additive field strength (with a few minor variables tossed in to keep the calculator modest).

Phase 1

Tom, like Roger, noted that these figures apply only to the search for maximum forward gain. They also note that anyone can verify this table with a little modeling. This is, in fact, what I did. I modeled a collection of 11 Yagis, each with a different gain ranging from a free space gain of 6.38 dBi to 13.47 dBi. (I actually modeled a number of others, but included only 11 in this collection so as 6not to lose sight of the progressions in a morass of excess data.) Besides models already in my own collection of models, I used the untapered (all too often called by the illogical name "monotapered") element dimensions of numerous Yagis provided in the Brian Beezley, K6STI, collection within YA, the Yagi evaluation program supplied with the *ARRL Antenna Book* in recent editions. The test frequency was 14.175, in the middle of 20 meters. Although the untapered element models are impractical to build, K6STI also provides in YA the recommended set of tapered element lengths.

Some of the higher gain antennas are also impractical from another standpoint: they are downward-frequency-upward-mechanical scalings from antennas for 15 and 10 meters. I doubt that most builders would attempt Yagis on 60' to 120' booms. However, the antennas show excellent characteristics.

The following table characterizes the basic data about each antenna's performance, both in free space, and at 70' and 140' (1 and 2 wl) above medium level homogenous earth. Feedpoint impedance is omitted, as it appears to play no role in stacking heights. All of the usual reservations about antenna models apply.

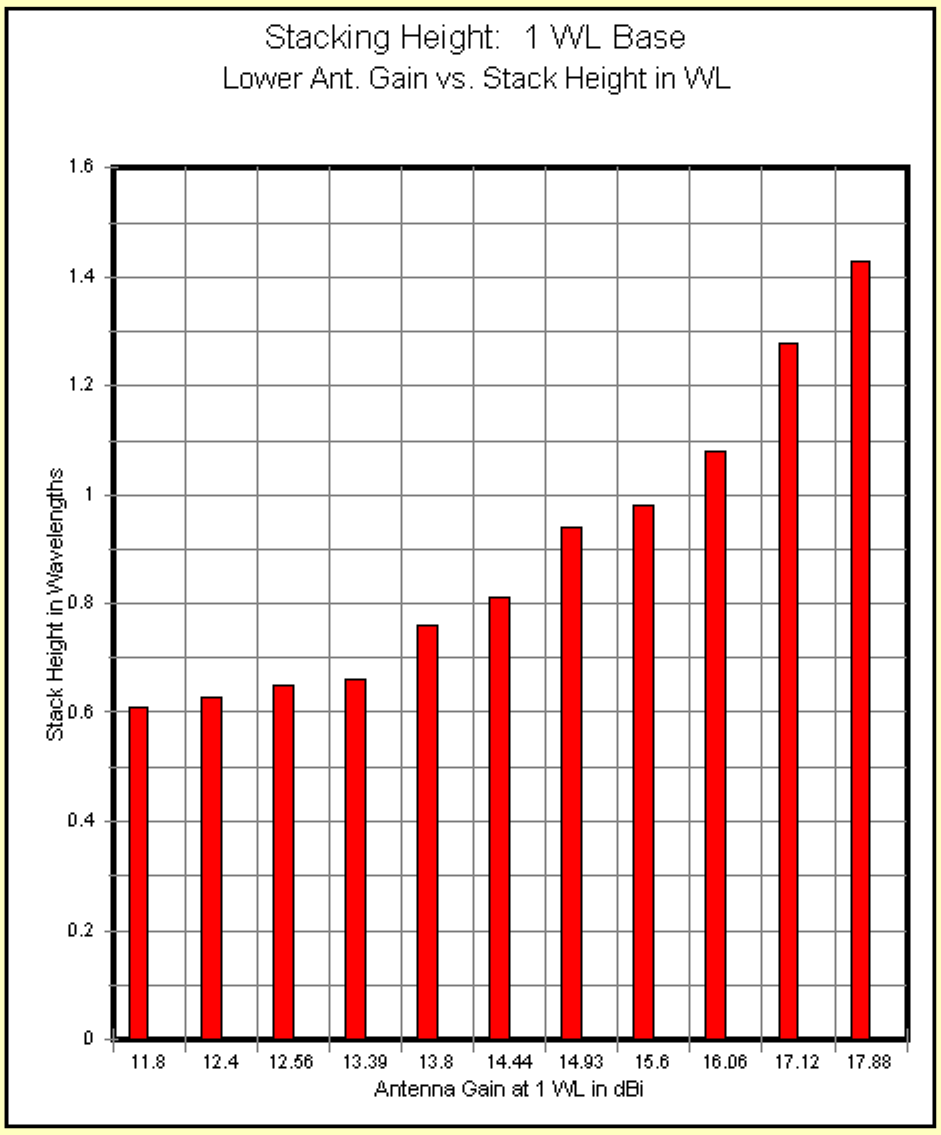
No. of Elements	Boom Ft	FS Gain dBi	FS F-B dB	70' Gain dBi	70' F-B dB	140' Gain dBi	140' F-B dB
2	10	6.38	11.15	11.80	12.71	12.12	11.84
3	24	7.02	21.07	12.40	20.15	12.75	20.72*
3	16	7.22	35.55	12.56	34.97	12.94	59.01**
3	24	8.11	27.28	13.39	25.05	13.81	25.61***
4	26	8.58	22.32	13.80	21.47	14.26	21.47
5	34	9.28	26.69	14.44	26.37	14.94	26.58
5	40	9.81	22.99	14.93	23.04	15.47	23.24
5	48	10.55	25.56	15.60	24.91	16.19	25.22
6	60	11.05	29.81	16.06	28.91	16.69	30.06
7	90	12.35	25.07	17.12	25.46	17.90	25.95
7	120	13.47	22.42	17.88	23.97	18.94	22.73

The 3-element Yagis marked with stars were used to see if boom length had any significant influence on stacking separation. Hence, a wide-band 24' boom model was contrasted to a shorter boom model with similar gain and a 24' boom model with higher gain. Boom length appears to exhibit an insignificant effect upon stacking separation.

Results of the modeling exercise form a reasonably ordered progression of the included gain values at nice intervals for comparison. However, the gain values do not form a linear set, so that the results are best presented as bar graphs without an artificial connecting line.

The progression is not absolutely orderly. However, the stacking spacing in terms of fractions of a wavelength required for maximum gain performance shows a close relationship to the gain over real ground of the lower antenna alone. The multiplier to derive the spacing in wavelengths ranges from about 0.05 times gain in dBi for the lowest gain beams to about 0.8 times gain in dBi for the highest gain Yagis. Moreover, as the gain of the individual antennas is increased and stacking space increases, so too does the range of space within which the maximum stack gain varies by only 0.01 to 0.02 dB, a difference from maximum that is too small to make the slightest difference to any operation. For example, at either the lower or higher altitudes, the stacks of the highest gain beams held their peak value for at least 5' or more. In contrast, maximum gain for the lower gain antennas peaked more sharply. In general, peaks with higher stacks were broader than peaks with lower stacks.

Separate "peaking runs" were made with the lower antenna at 1 wl (70') and with the lower antenna at 2 wl (140'). (The actual wavelength at 14.175 MHz is 69.38774' long, a little less than 1% shorter than the rounded height used.) Figure 1 shows the required separation for maximum gain with the antennas ordered in accord with their individual gain at 70'.

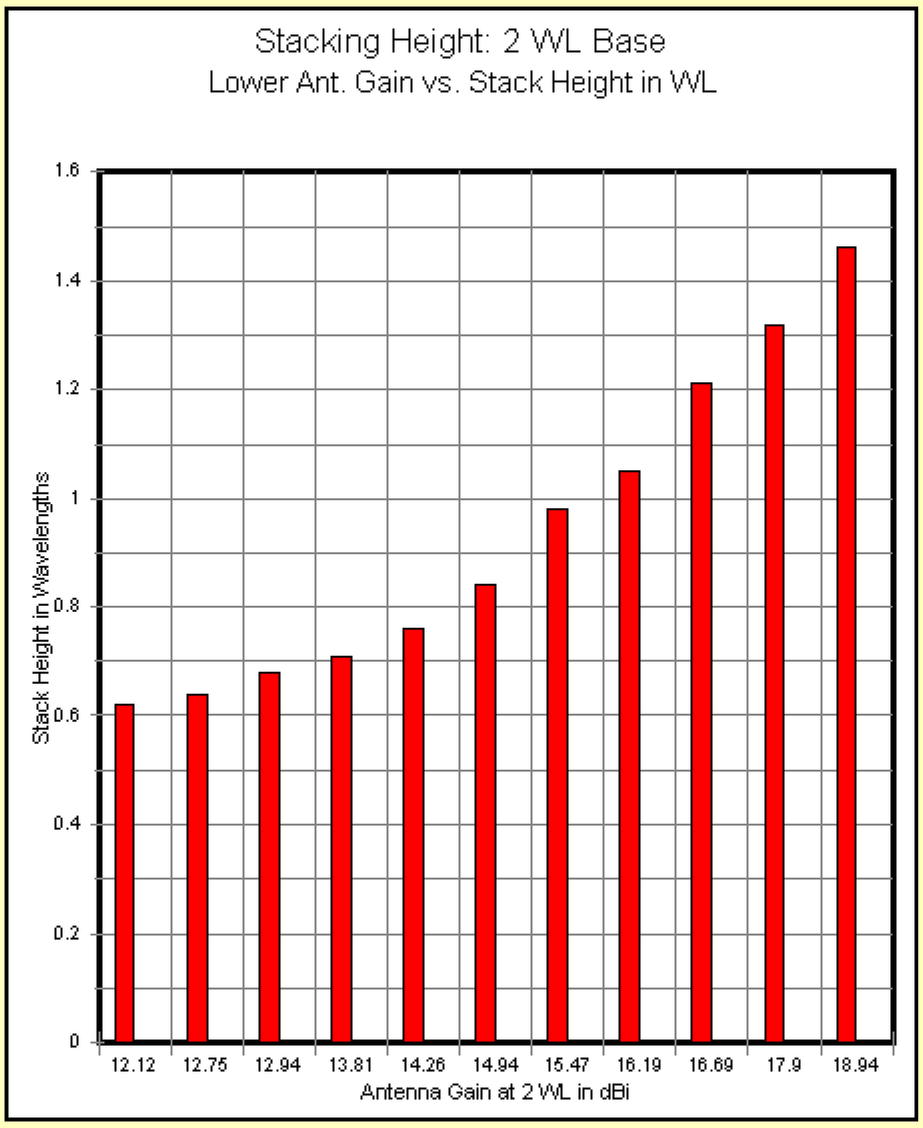


The following table shows the relationship of individual gain to maximum gain attained in models and the range of separation for peak stack gain. also shown is the front-to-back ratio for the stack.

No. of Elements	**Boom Ft**	**70' Gain dBi**	**70' F-B dB**	**Stack gain dBi**	**Stack F-B dB**	**Separation in WL**
2	**10**	**11.80**	**12.71**	**15.02**	**13.58**	**.59-.62**
3	**24**	**12.40**	**20.15**	**15.36**	**12.15**	**.62-.63**
3	**16**	**12.56**	**34.97**	**15.59**	**15.69**	**.65**
3	**24**	**13.39**	**25.05**	**16.21**	**17.56**	**.65-.68**
4	**26**	**13.80**	**21.47**	**16.58**	**23.23**	**.76**
5	**34**	**14.44**	**26.37**	**17.17**	**24.19**	**.81-.82**
5	**40**	**14.93**	**23.04**	**17.53**	**17.56**	**.94-.95**
5	**48**	**15.60**	**24.91**	**18.11**	**37.34**	**.95-1.01**
6	**60**	**16.06**	**28.91**	**18.46**	**28.78**	**1.07-1.11**
7	**90**	**17.12**	**25.46**	**19.49**	**24.00**	**1.25-1.31**
7	**120**	**17.88**	**23.97**	**20.36**	**24.33**	**1.40-1.46**

Single separation values do necessarily indicate especially sharper peaks than those covering a span of 2-3% of a wavelength, since often the next lower value of gain holds longer for the single value peaks. However, the general trend of having wider latitude of separation selection while still achieving maximum gain for Yagis with higher gain holds good.

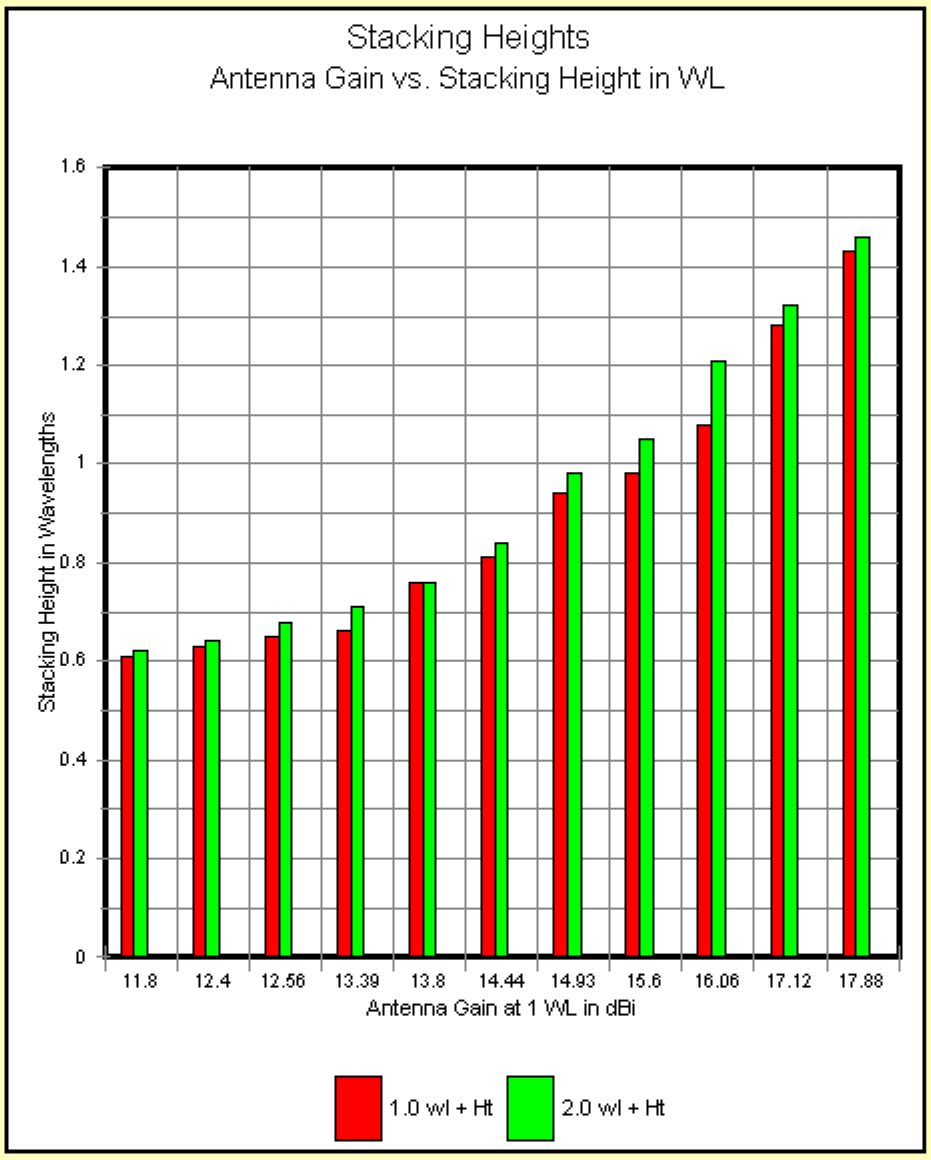
A second set of runs were made with the lower antenna set at 140' (2 wl). The run is significant in that it shows some small departures from the lower level runs. Most significant are some small but definite variations in the progression of spacing required. Equally significant is the fact that for beams of nearly the same gain, the required separation for maximum gain was greater if the beams were higher than if lower. Compare, for example, the 7-element beams. The 90' boom model has, at 140', virtually the same gain as the 120' boom model at 70' up. However, the higher array demands about 1.42 wl spacing, while the lower one requires about 1.32 wl spacing. Similar results can be found elsewhere in the following figure and table.



No. of Elements	Boom Ft	140' Gain dBi	140' F-B dB	Stack gain dBi	Stack F-B dB	Separation in WL
2	10	12.12	11.84	15.60	12.66	.60-.63
3	24	12.75	20.72	15.95	11.88	.63-.63
3	16	12.94	59.01	16.21	15.75	.68-.69
3	24	13.81	25.61	16.87	17.69	.71-.72
4	26	14.26	21.69	17.27	22.86	.75-.78
5	34	14.94	26.58	17.89	24.72	.84
5	40	15.47	23.24	18.30	19.04	.97-.99
5	48	16.19	25.22	18.96	32.07	1.05
6	60	16.69	30.06	19.41	29.68	1.17-1.25
7	90	17.90	25.95	20.55	24.66	1.28-1.38
7	120	18.94	22.73	21.51	23.94	1.43-1.51

An interesting side note on the data stream is that the wider the separation of beams required for maximum stack gain, the more likely the stack is to hold most or all of the front-to-back ratio of the individual antennas.

Yagis are as full of variables as they are full of regularities. Anomalies in the progressions appear at various points, especially between the curves for the 1-wl base and the 2-wl base. The following graph, using the 1 wl gain as the key to the models, parallels the separation requirements suggested by models for each antenna. Some, but by no means all, of the minor anomalous behavior of the progressions can be attributed to the fact that I took the simple mean between extremes for the range of separation giving peak gain as the graphed value. However, as the graph shows, other variations remain. None of the variations is sufficient to result in any detectable difference in performance using the stack separations suggested by the models or from using figures developed to create smoother curves.

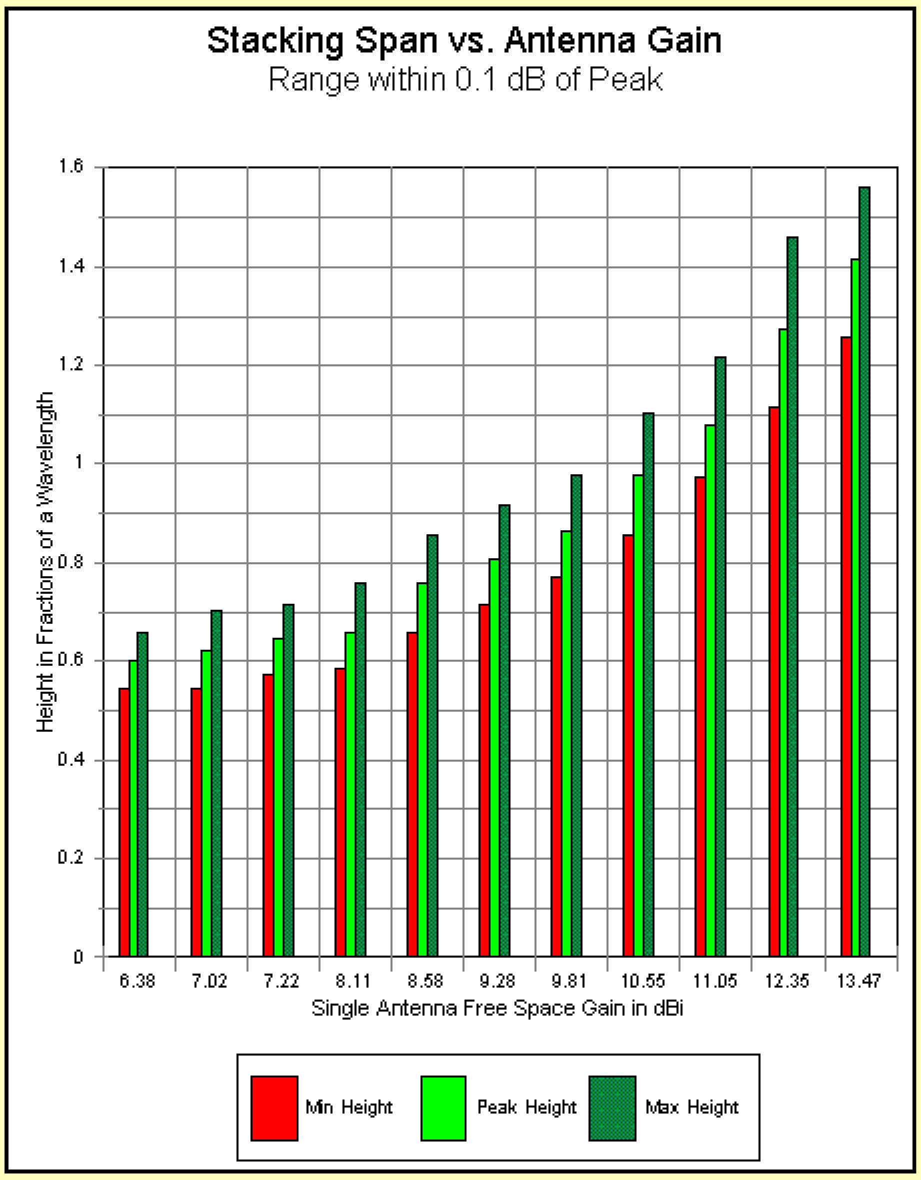


Phase 2

The peak gains of the antennas and their required stacking heights approximate those of the table early in these notes. However, there are minor variations that led me back to the models for a second look. Suppose we look at the gain figures in another way. Instead of seeking only the peak gain, let's also set a limit for a range of heights such that the antenna pair gain is down no more than 0.1 dB from the peak. This gives us a chance to choose a near maximum gain and to select a height with hopefully a better front-to-back ratio, if that is possible. Only the data for antenna pairs with the lower antenna at 70' up at 14.175 MHz were examined in detail. The following two tables and graphs summarize the findings.

No. of Elements	Boom Ft	Min Ht. feet	Min Ht. wl	Peak Ht. feet	Peak Ht. wl	Max Ht. feet	Max Ht. wl
2	10	38	.54	42	.60	46	.66
3	24	38	.54	43.5	.62	49	.70
3	16	40	.57	45	.64	50	.72
3	24	41	.59	46	.66	53	.76
4	26	46	.66	53	.76	60	.86
5	34	50	.72	56.5	.81	64	.92
5	40	54	.77	60.5	.87	68	.97
5	48	60	.86	68	.97	77	1.10
6	60	68	.97	75.5	1.08	85	1.22
7	90	78	1.12	89	1.27	102	1.46
7	120	88	1.26	99	1.42	109	1.56

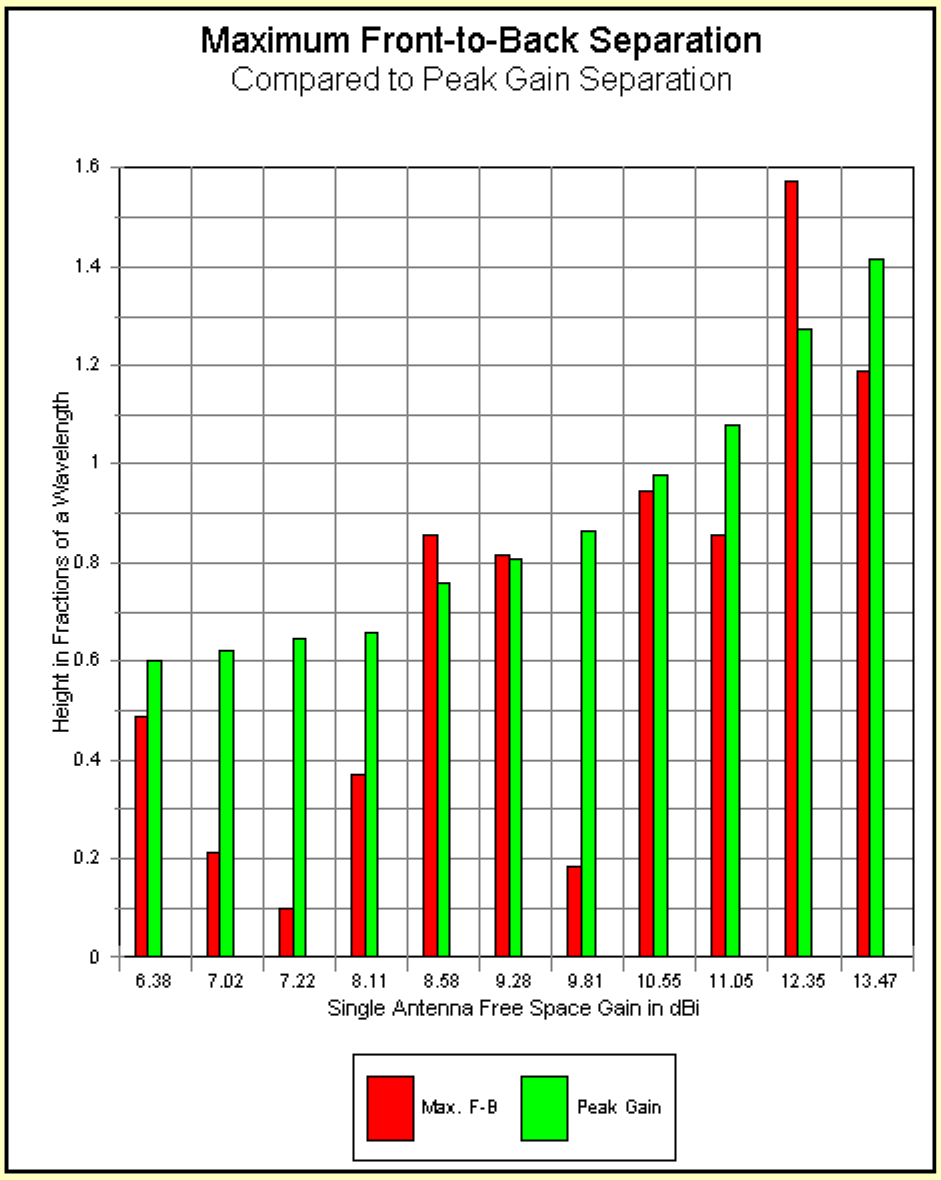
Here is the range for each figure in graphical form.



The second table lists the gain and front-to-back ratios of the stacked pairs at the minimum and maximum heights for 0.1 dB down from peak gain, along with the heights for maximum front-to-back ratio and the gain and ratio achieved at that height. Peak gain figures for both gain and front- to-back ratio are available in an earlier table.

No. El.	**Boom Ft**	**Min Ht. gain dBi**	**Min Ht F-B dB**	**Max Ht gain dBi**	**Max Ht F-B dB**	**Max F-B Ht. f/wl**	**Max F-B gain dBi**	**Max F-B F-B dB**
2	**10**	**14.93**	**18.66**	**14.94**	**10.90**	**34 / .49**	**14.65**	**27.15**
3	**24**	**15.25**	**13.75**	**15.25**	**12.27**	**15 / .21**	**13.64**	**30.35**
3	**16**	**15.48**	**15.34**	**15.50**	**16.80**	**7 / .10**	**12.94**	**20.54**
3	**24**	**16.12**	**18.16**	**16.11**	**20.01**	**26 / .37**	**15.19**	**26.52**
4	**26**	**16.47**	**18.81**	**16.48**	**27.48**	**60 / .86**	**16.48**	**27.48**
5	**34**	**17.06**	**23.01**	**17.07**	**23.73**	**57 / .82**	**17.17**	**24.19**
5	**40**	**17.43**	**16.15**	**17.43**	**19.45**	**13 / .19**	**14.96**	**25.50**
5	**48**	**18.00**	**30.02**	**18.02**	**28.39**	**66 / .94**	**18.11**	**39.57**
6	**60**	**18.37**	**29.94**	**18.36**	**29.59**	**60 / .86**	**18.04**	**31.49**
7	**90**	**19.39**	**23.84**	**19.39**	**26.23**	**110 /1.57**	**19.23**	**27.58**
7	**120**	**20.26**	**24.55**	**20.26**	**23.69**	**83 /1.19**	**20.13**	**24.62**

The following graph may make the data clearer by setting the maximum front- to-back separations next to the peak gain separations.



From the data, several things are clear. First, there is not an exact agreement with the initial tables supplied me and the findings from these models, although the ranges are quite close indeed. The differences may stem from several sources. 1. The model antennas used are different in each case, so no exact correlation can be drawn between any two antennas, chosen one each from the supplied and the developed results. 2. Modeling programs will differ in their gain figures, both as a result of differences in the modeling machine and as a consequence of the ground calculations used by the program. MININEC, for example, uses a simplified ground approximation, while NEC-2 and -4 use the more accurate Sommerfeld-Norton calculation. Hence, the differences in the recommended ranges are not significant.

Second, the ideal maximum front-to-back ratio separation height between two identical antennas may be above, below, or within the range of maximum gain. In some cases, the front-to-back ratio of a stack is acceptable at one of the maximum gain heights, even if not fully maximized. In some other cases, the front-to-back ratio may be low enough at a maximum gain height to adversely affect operations. There is no easy correlation between the maximum front-to-back height and any single or pair of antenna properties, including number of elements, boom length, feedpoint impedance, or the front-to-back ratio of a single antenna.

Phase 3

It pays to look carefully at the elevation patterns of stacks. The nulls are just as important as the lobes. Moreover, the elevation angle of maximum radiation of a stack is always lower than the take-off angle of the top beam alone, since the stack is a single array with a composite pattern equivalent to a single planar antenna at some intermediate height. Moreover, although the effect may be small in a well-designed stack, the use of one of the two antennas while the other is passive will alter at least slightly the figures for the individual antennas.

To illustrate these points, I set up a stack of identical 4 element Yagis from my collection of models for mid-band on 15 meters. For this beam design (about 8.6 dBi forward gain in free space), a separation of 35' yielded nearly maximum forward gain for the pair while preserving a better than 20 dB front-to-back ratio. The feed Z was about 25 - 28 Ohms.

I placed the beams at 100' and 135' up. Here are the figures for these beams when each is the only beam on the tower:

Height	**Gain**	**To angle**	**Vert B/W**	**2nd lobe**	**F-B**
feet	**dBi**	**degrees**	**degrees**	**degrees**	**dB**
100'	**14.27**	**6.4**	**6.5**	**20.0**	**23.08**
135'	**14.36**	**4.8**	**4.8**	**14.7**	**21.92**

Now let's stack the beams.

Height	**Gain**	**To angle**	**Vert B/W**	**2nd lobe**	**F-B**
feet	**dBi**	**degrees**	**degrees**	**degrees**	**dB**
Both	**17.33**	**5.4**	**5.5**	**16.5**	**20.77**
135'	**14.28**	**4.8**	**4.7**	**14.7**	**20.81**

As you can see, the figures for the 135' only and the 135' w/100' passive are very slightly different, but operationally insignificantly different in this arrangement. The feed Zs also differ in resistance by about 1 Ohm (4%).

Moreover, the TO angle of the main lobe is lower for the stack than for the top beam, either only or w/100' passive. You can repeat this exercise with innumerable beams in well arranged stacks.

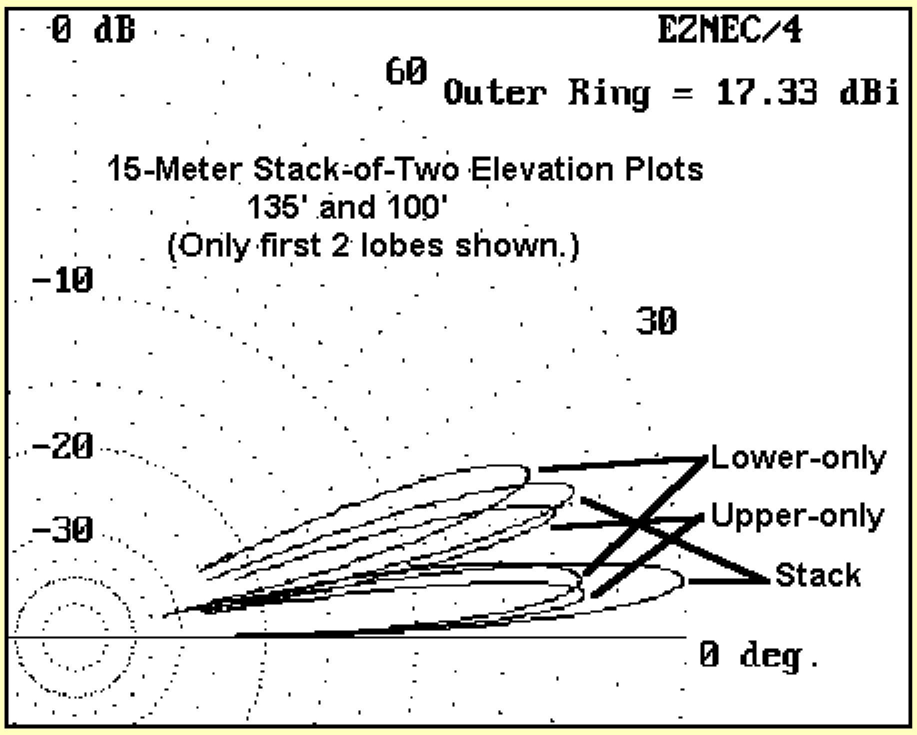
Now compare the "Both" entry with the following:

Height	**Gain**	**To angle**	**Vert B/W**	**2nd lobe**	**F-B**
feet	**dBi**	**degrees**	**degrees**	**degrees**	**dB**
120'	**14.33**	**5.4**	**5.4**	**16.7**	**20.80**

The stack fed in phase is angularly (but not gain) equivalent to a single beam at 120', about 57% of the distance of separation.

For all of these exercises, I have let the software calculate patterns to 0.1 degrees so that differences are not washed out, even if they do not make a big or even a little difference operationally. Since the two beams in this modeled stack are about 2.15 wl and 2.9 wl up respectively, differences become small, and comparative modeling should go to smaller angular resolution in order to see differences that are at least mathematically significant.

But now let's explore those 2nd lobes I added to the scheme. These lobes are in fact not down by much relative to the main lobe. However, between the lowest lobe and the next one up is a null (the one to which I originally referred) that may be 20 dB or more deep at its pinpoint extreme. The null is at about 9.8 degrees for the 135'-only operation, 11.0 degrees for the stack, and 13.2 degrees for the 100' only operation. On some occasions, the incoming signal angle may be well into one of these nulls, but never all three simultaneously--because the nulls are exceptionally sharp.



It is easy to see the importance of having the ability to switch among arrangements in order to change the null position--or more operationally, to change the lobe position to catch the signal. A couple of degrees difference in the lobe angle (either the lowest or the 2nd) might indeed make a 15-20 dB signal strength difference. Moreover, to fill the null completely, a stack of 3 might be even more useful, since it would offer the use of beams 1-3, 1-2, 2-3, and the individual beams.

Of course, when beams are lower in height relative to a wavelength at the operating frequency, the lobes are vertically wider. This variation in lobe size as well as all of the variables of individual beam placement in the stack for the many possible installations makes this an exercise each stacker should carry out for his or her own installation.

The upshot is that a stack is as likely to miss some action as the top beam alone or the bottom beam alone, since at a given moment, the particular path of interest may fall into the null between the first and second lobes. If you plan a stack, especially one with a top height well over 2 wl (and here approaching 3 wl), switching capability may be the key to using the stack effectively.

Now the usual disclaimer: modeling via NEC is over flat ground with no bumps or clutter. For other types of terrain, the angles may change, but the progressions will not. Use either the N6BV or the K6STI terrain software to translate these figures into those which better fit your situation.

Conclusion

The achievement of maximum gain from a stack of two identical monoband Yagis is straightforwardly determined. The traditional range charts are very close for almost any antenna selected having a similar free space gain.

However, selection of antennas for a stack requires close attention to design for maximum results with respect to gain, take-off angle, and front-to-back ratio. Given the diverse results for both 3-element beams and 5-element beams--and remembering that a different arrangement of elements on a similar boom length might give different front-to-back results in a stack-- the only safe course for a stacker is to model or have modeled a range of potential antenna pairs. And, if one goes to this trouble in order to maximize the cost-benefit ratio, one might as well also subject the proposed stack to either N6BV's or K6STI's terrain analysis programs.

Considering the size of the investment needed for a stack and all of the supplementary mechanical and material supplies to make the stack work, selecting antennas that promise both high gain and excellent front-to-back properties may not significantly raise the cost. Such careful selection, however, may improve the operating capabilities of the stacked array.

Go to Amateur Radio Page